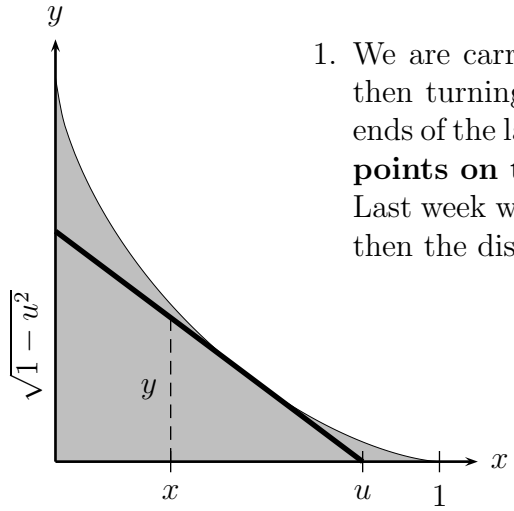
Douglass Houghton Workshop, Section 1, Wed 12/07/16

Worksheet Panda Bear



1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last week we found that if the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is

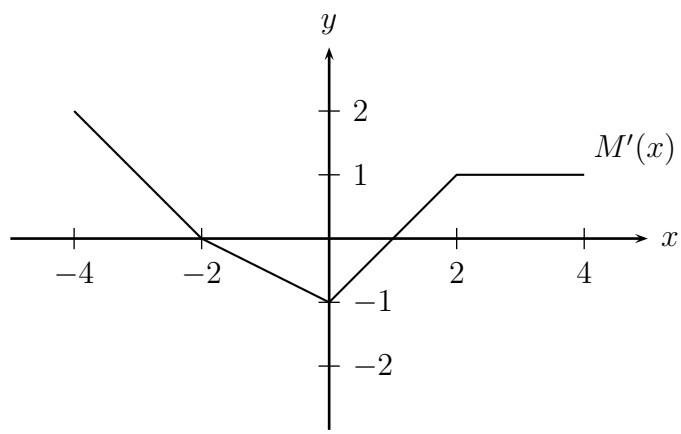
$$y = \frac{u-x}{u}\sqrt{1-u^2}$$

   and on Monday we found that y is maximized when $u = \sqrt[3]{x}$.

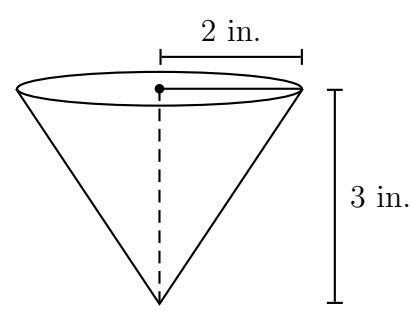
   (a) So for a fixed x, what is the maximum value of y, as the ladder moves?

   (b) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

2. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.



3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



4. (This problem appeared on the Winter, 2015 Math 115 Final Exam) For nonzero constants a and b with $b > 0$, consider the family of functions given by

$$f(x) = e^{ax} - bx.$$

Note that the derivative and the second derivative of $f(x)$ are given by

$$f'(x) = ae^{ax} - b \qquad \text{and} \qquad f''(x) = a^2 e^{ax}.$$

(a) Suppose the values of a and b are such that $f(x)$ has at least one critical point. For the domain $(-\infty, \infty)$, find all critical points of $f(x)$, all values of x at which $f(x)$ has a local extremum, and all values of x at which $f(x)$ has an inflection point. (Note that your answer(s) may include the constants a and/or b.)

(b) Which of the following conditions on the constant a guarantee(s) that $f(x)$ has at least one critical point in its domain $(-\infty, \infty)$?

(i) $a < 0$ (ii) $0 < a < b$ (iii) $b < a$

(c) Find exact values of a and b so that $f(x)$ has a crtical point at $(1, 0)$.

5. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

(a) $P'(2) = 55$

(b) $\int_5^{10} P'(t)\,dt$

(c) $P^{-1}(690)$

(d) $\frac{1}{5}\int_5^{10} P(t)\,dt$

6. (Fall 2008) This problem was a smörgåsbord:

(a) If $f(x)$ is even and $\int_{-2}^{2} (f(-x) - 3)\,dx = 8$, find $\int_0^2 f(x)\,dx$.

(b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

(c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_3^{27} R(t)\,dt$ means in this context.

(d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

(e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \le 1 \\ 6\sin(\pi(x - 0.5)) + k & \text{if } x > 1 \end{cases}$$