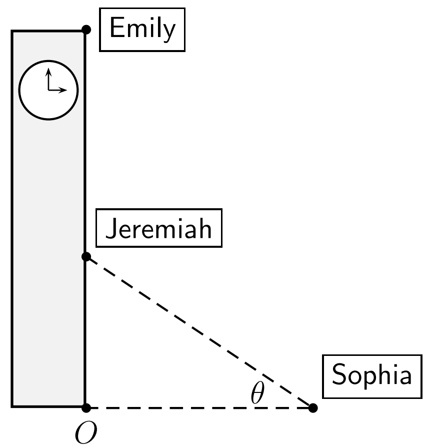
Douglass Houghton Workshop, Section 1, Mon 12/05/16

Worksheet Once More, unto the Breach



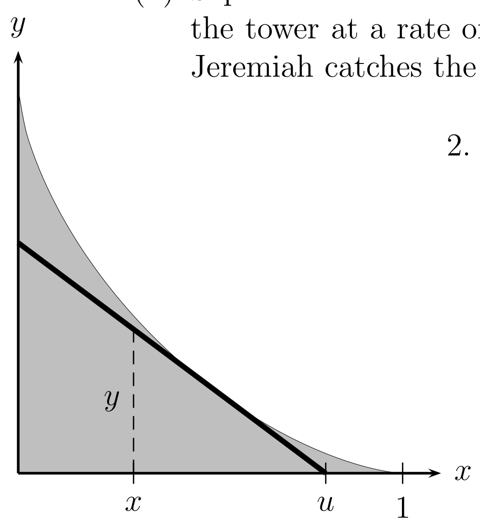
1. (Adapted from a Winter, 2005 Math 115 exam) One day Emily notices that the door to the Burton Tower carillon has been left open. She decides to barricade herself in the tower and sing songs from The Sound of Music.
   Jeremiah has now fully recovered from breaking his leg on April 15th, and so the University hires him to climb the tower and recapture it, in the spirit of Frozone. On the ground below, 30 feet away, Sophia watches and interprets the scene in dance, as she used to interpret the shapes of graphs in math class. She looks up at an angle θ to see Jeremiah.

   (a) Find a formula for the rate of change of Jeremiah's distance from the point O with respect to θ.

   (b) If the distance from point O to Emily is 200 ft. and Jeremiah is climbing at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Jeremiah is halfway up?

   (c) When Jeremiah is halfway up, Emily drops a rope down to him. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Jeremiah catch it, and what is its speed when he does?

   (d) Sophia watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of her gaze changing when Jeremiah catches the rope?



2. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is

   $$y = \frac{u - x}{u}\sqrt{1 - u^2}.$$

   What value of u maximizes y? (Keep x fixed!)

3. (This problem appeared on a Winter, 2004 Math 115 Final Exam.) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$, measured in songs/hour where $t = 0$ corresponds to 5 pm. Explain the meaning of the quantity $\int_0^5 m(t)\, dt$.

4. Suppose $\int_4^9 (4f(x)+7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

5. Suppose $\int_a^b f(x)\,dx = 2$ and $\int_a^b g(x)\,dx = 4$. Evaluate the following expressions, if possible. Assume that all functions are continuous on the interval $[a,b]$.

   (a) $\int_a^b (g(x))^2\,dx - \left(\int_a^b g(x)\,dx\right)^2$

   (b) $\int_{a+2}^{b+2} f(x-2)\,dx$

   (c) $\int_a^b (f(x)g(x))\,dx$

   (d) $\int_b^a (g(x))\,dx$

6. (This problem appeared on the Winter, 2004 Math 115 Final Exam) It is estimated that the rate people will visit a new theme park is given as

$$r(t) = \frac{A}{1+Be^{-0.5t}}$$

   where A and B are both constants and $r(t)$ is measured in people per day, and $t=0$ corresponds to opening day.

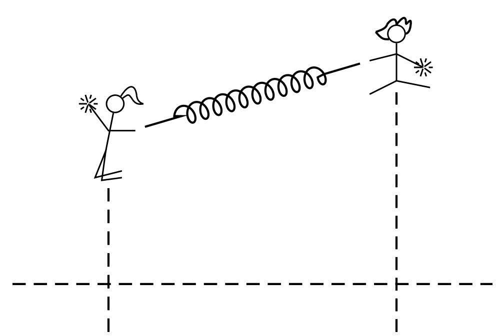
   (a) Write an integral that gives the total number of people visiting the park in the first year it is open. Do not try to evaluate the integral!

   (b) Suppose that $A=100$ and $B=5$. Given that

$$\frac{d}{dt}\left(2A\ln(1+Be^{-0.5t}) - 2A\ln(Be^{-0.5t})\right) = \frac{A}{1+Be^{-0.5t}},$$

   use the First Fundamental Theorem of Calculus to evaluate how many people visit the park during the first year it is open. Make sure you clearly indicate your use of the theorem.

7. Jessica and Ajené are performing a routine in a STUNT cheerleading competition. They are both thrown straight up into the air from positions 15 feet apart, and they hold a slinky between them.



   (a) How fast is the slinky expanding when Ajené is 13.96 feet above the ground and rising at 1.6 ft/sec, while Jessica is 7 feet above the ground and falling at 14 ft/sec?

   (b) Jessica is thrown upward with an initial velocity of 18 ft/sec at time 0, and Ajené is thrown .3 seconds later with initial velocity 24 ft/sec. Both start 5 feet above the ground and are subject to the acceleration of gravity, which is $-32\,\text{ft/sec}^2$. Find formulas for their heights at time t.

   (c) Find the maximum and minimum length of the slinky between the time Ajené lifts off to the time Jessica is caught by her teammates.