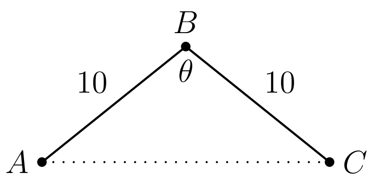
Douglass Houghton Workshop, Section 1, Mon 11/28/16

Worksheet May the Road Rise to Meet You

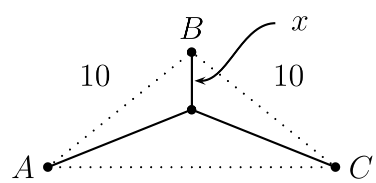
1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible λ's.

   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the λ-shaped network to the right.

   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 10. Put the graphs on the board.



   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Write the following sums in sigma ($\sum$) notation.

   (a) $1 + 2 + 3 + 4 + \cdots 10$

   (b) $1 + 2 + 3 + 4 + \cdots + n$

   (c) $3 + 5 + 7 + 9 + \cdots + 21$

   (d) $4 + 9 + 16 + 25 + \cdots + 100$

   (e) $2.3 + 2.8 + 3.3 + 3.8 + 4.3 + 4.8 + \cdots + 10.3$

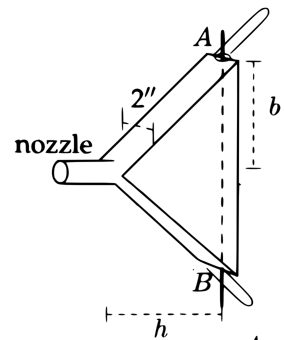
   (f) $f(a_1) + f(a_2) + f(a_3) + \cdots + f(a_n)$

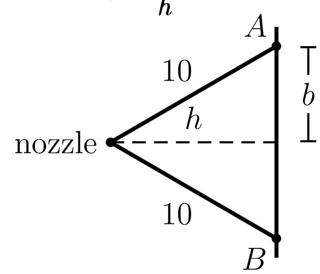
3. Consider the function $f(x) = x^x$.

   (a) It's neither a power function (ax^b) nor an exponential (ab^x). Nevertheless, find its derivative. Hint: rewrite it in the form $e^{u(x)}$ for some function u.

   (b) What is the minimum value that f takes on? (Check with your calculator, but find the answer with calculus.)

4. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



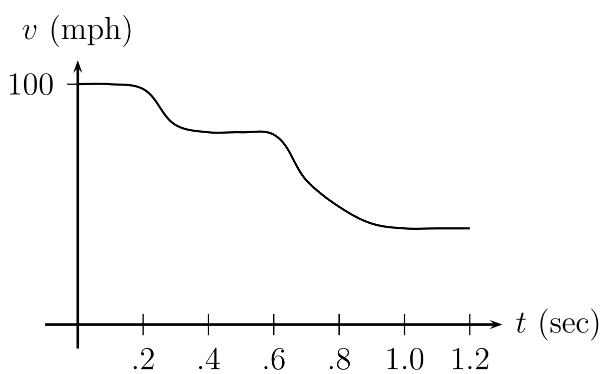


5. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

   (a) Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

   (b) How often would you have to weigh the baby to get an estimate guaranteed to be accurate to within $\frac{1}{4}$ poind?



6. After talking to the baseball the way his hero, Mark "The Bird" Fidditch, used to when he pitched for the Detroit Tigers, Jack throws it toward the outfield. The ball passes through various air currents along the way, and its horizontal velocity, in miles per hour, is given by the graph to the left. Estimate the distance the ball travels in the first 1.2 seconds.