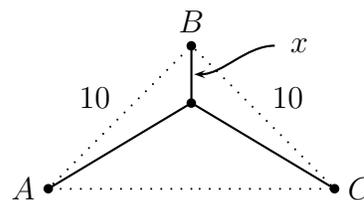


Worksheet Love All, Trust a Few, Do Wrong to None

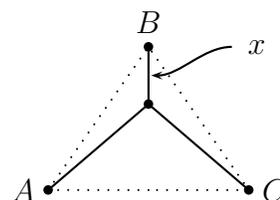
- We've been working on the problem of finding the shortest road network between three cities in the plane.

In the case we considered, the three cities were at the corners of a 45° - 45° - 90° triangle with legs 10 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 20. But by constructing a Y-shaped network like the one at the right, we found



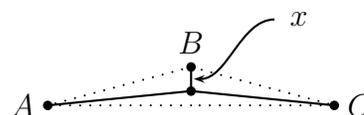
- The length of the network is $x + 2\sqrt{100 - 20x \cos(45) + x^2}$.
- We can improve from the simple 2-road solution ($x = 0$, length = 20) by increasing x . For instance, when $x = 5$, the network has length 19.74.

- Consider the case where the triangle is still isosceles and the legs still have length 10, but the angle at B is 70° . Write a formula for the length of the network.



- Can you find a value of x which beats the 2-road solution ($x = 0$, length = 20)?

- Now suppose the vertex angle is very obtuse—say 150° . Find a formula for the length of the network.



- Can you beat the 2-road solution in this case?

- Suppose the vertex angle is θ . Write a formula for the length of the network.

- (Adapted from a Fall, 2001 Math 115 final exam) Kaila starts a business selling mushrooms that produce ethanol. She packages the mushrooms in long-necked yellow and brown jars with the slogan “These mushrooms will make you feel as tall as a giraffe!” Her cost and revenue functions (in dollars) are

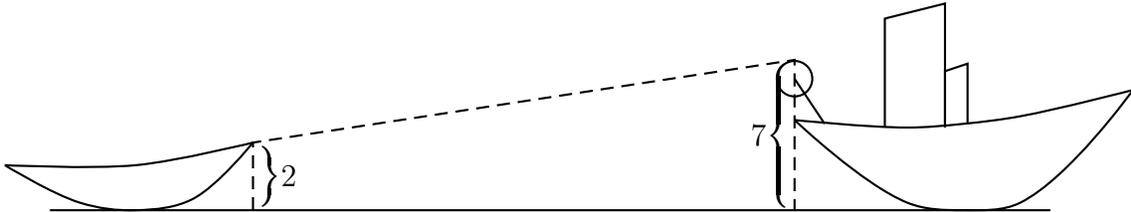
$$C(q) = 2000 + 40q \quad \text{and} \quad R(q) = 300q^{.75}$$

where q is the number of jars produced.

- What is the product's fixed cost?
- Last year, Kaila produced 2400 jars of mushrooms. What was her profit?
- Find formulas for the marginal cost and marginal revenue, and evaluate at $q = 2400$.
- Kaila would like to increase production and do better this year. Based on the marginal cost and marginal revenue *at this point* ($q = 2400$), explain whether her strategy is sound.

3. (Adapted from a Fall, 2006 Math 115 Final Exam.) Adrianna's final musical production at her high school was such a success that the entire cast takes a boat ride on Saginaw Bay to celebrate. Unfortunately, their boat breaks down and requires a tow.

A cable is attached to the front of the boat 2 meters above the water. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of a tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



- (a) At what rate is the length of the cable between the two boats changing?
- (b) How fast is the small boat being pulled forward when it is 10 meters away from the tugboat?
4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Ariel. At the end of every day in November she uploads her manuscript to a website (<http://tinyurl.com/worlds-over-again>), which counts how many words she has written. Here are her counts from last year, rounded to the nearest hundred:

Nov.	Count								
1	0	7	0	13	13800	19	21300	25	21300
2	0	8	0	14	13800	20	21300	26	21300
3	0	9	0	15	18400	21	21300	27	21300
4	0	10	0	16	20300	22	21300	28	21300
5	0	11	12700	17	21300	23	21300	29	38700
6	0	12	12700	18	21300	24	21300	30	51100

- (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x . (So, for instance, $W(14) = 13800$.) Assume that each day Ariel writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the week from November 11 to 18 (x from 11 to 18).
- (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 11 to 18.
- (c) Now consider the function $F(t)$, which is the area between the line $x = 11$, the line $x = t$, the x -axis, and the graph of $w(x)$. Make a table of values showing $F(11), F(12), \dots, F(18)$. What do you notice? Explain this result.