

Worksheet Kangaroo

1. The three cities in the pictures below are at the corners of an 45° - 45° - 90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.



(Say, A is Ann Arbor, B is Lansing, and C is Battle Creek.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C . That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B , then building a road north from there to B .

- Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?
 - Calculate the total length of the new network in terms of x . Hint: “Law of cosines”.
 - Can you find a value of x which will produce a shorter network than the simple proposal?
2. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing “Hail to the Victors” has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

- How long should the band play to maximize the number of points Michigan scores?
- The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

- What will be the score of the game for the case you found in part (b)?

3. In “The 12 days of Christmas”, a certain poultry-afficianado receives a number of gifts from her true love:

Day 1: A partridge in a pear tree. How to get it down?

Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

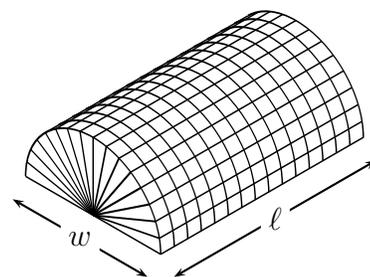
Day 3: 3 French hens, 2 more turtle doves, and another partidge.

...

Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

- (a) If item 1 is “partridge”, item 2 is “turtle dove”, etc., then write a formula for the total number of item n 's received.
- (b) Of which item does Mr. Truelove send the most? (Solve using calculus.)
4. (This problem appeared on a Winter, 2008 Math 115 Exam.)
- (a) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f ?
- (b) Find all critical points, local maxima, and local minima of f .
- (c) Which of the local maxima and minima are global maxima / minima?

5. (From a Winter, 2011 Math 115 exam) A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length l , is $1 + 0.001\ell$ per square foot.



- (a) Write an equation for the cost of a hoophouse in terms of ℓ and w .
- (b) Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.
6. Kayla drives east on the Ohio Turnpike to a softball tournament She takes a ticket out of the machine in Toledo, and then turns it in at the toll booth near Columbus. Along with her change, the State Trooper in the toll booth hands Kayla a speeding citation, and says that he *knows* Kayla was going exactly 75 mph at some point on her trip. How does the Mean Value Theorem tell the trooper that?