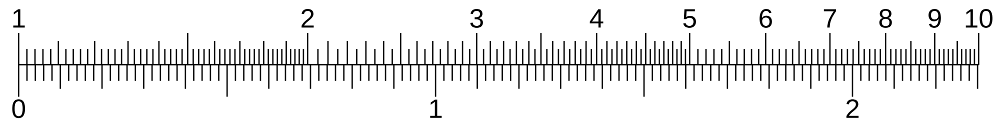
Douglass Houghton Workshop, Section 1, Wed 9/28/16

Worksheet Gone are the Days of Summer

1. What does this picture represent?



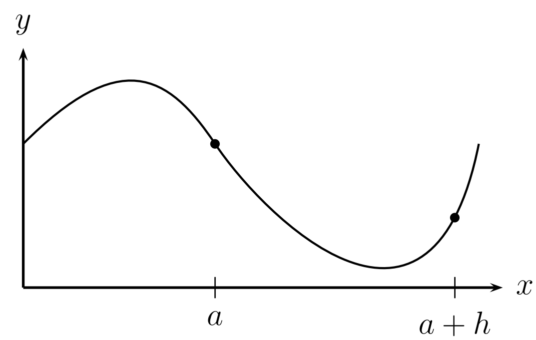
2. Explain how to use two rulers to add numbers.

3. Explain how a slide rule is able to multiply two numbers.

4. (This problem appeared on a Fall, 2004 Math 115 exam) For this problem, f is differentiable everywhere.

   (a) Write the limit definition of the derivative of the function f at the point a.

   (b) On the graph below, show how the rate of change of f between $x = a$ and $x = a+h$ is related to the derivative at the point a. Give a brief explanation of your illustration including how the limit as $h \to 0$ is demonstrated in your picture.



   (c) Write the limit definition for $f'(2)$ if $f(x) = e^{\sin 2x}$. [You do not need to find the limit or approximate $f'(2)$.]

   (d) (Added for DHSP) Oh, heck, go ahead and approximate $f'(2)$. Use your calculator, but not any fancy calculus features that it might have.

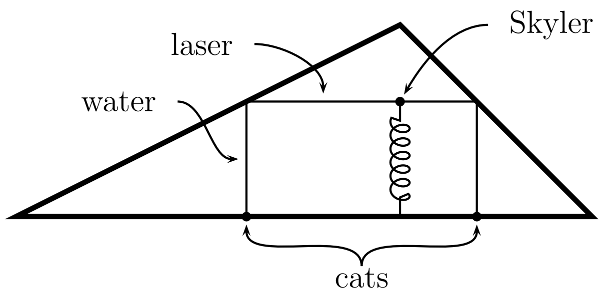
5. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

6. (This problem appeared on a Winter, 2009 Math 115 Exam) Air pressure, P, decreases exponentially with the height, h, in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

   (a) On top of Mount Denali, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

   (b) Determine $P^{-1}(0.7)$. Include units!

7. Imagine that Skyler is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Skyler goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Skyler's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Skyler's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Skyler is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Skyler's height off the floor.

8. Prove that it's possible to make a fair 5-sided die. Hint: do the previous problem first.

9. Last time we investigated a rule for how a population of mice might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 200$	800	Yes

An *equilibrium* is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is *stable* if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is *unstable*.

   (a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

$$P(n+1) = .4P(n) + 600 \qquad P(n+1) = -1.3P(n) + 460$$
$$P(n+1) = 1.1P(n) - 330 \qquad P(n+1) = P(n) + 300$$
$$P(n+1) = -.5P(n) + 1200 \qquad P(n+1) = -P(n) + 300$$

   (b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.