Douglass Houghton Workshop, Section 1, Mon 9/26/16

Worksheet Friends, Romans, and Countrymen

1. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

2. Gianna is studying a population of mice on the prairies of Illinois. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2016, $P(1)$ is the population 1 year later, etc.

   (a) Make up a (short) story about mice that yields that formula as the result.

   (b) Suppose there are 320 mice in 2016. What will happen in the long run?

   (c) Suppose instead that there are 800 mice in 2016. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

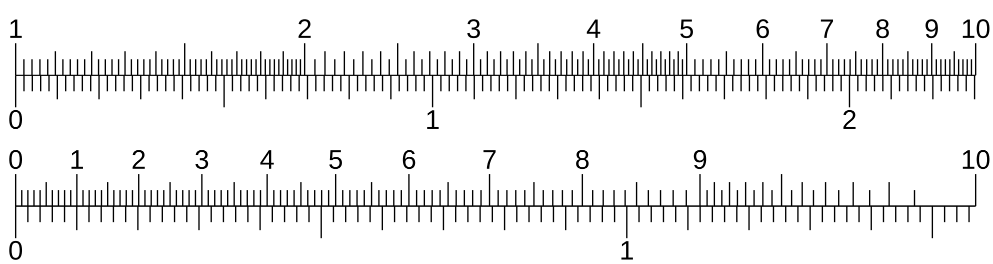
   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

3. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 200.$$

4. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two fish scenarios has a stable equilibrium?

5. What's the deal with these pictures? What are they good for?



6. Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.

7. (This problem appeared on a Winter, 2012 Math 115 exam. Really!) Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

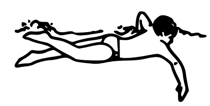
   (a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

   (b) According to your function, will the hull of the ship hit the reef? Explain.

   (c) (Added for DHSP) Draw a graph of the ship's height above the reef, versus time.

8. Fernando has noticed that his tastes changed over the last year. A year ago he spent about 15 hours a week swimming, and 10 hours taking apart computers. Gradually school took over his life, and though there have been some ups and downs in his schedule, the general trend is that he's spent less time per week on both. Now, 52 weeks later, he spends only 3 hours a week swimming and 5 hours a week taking apart computers.

   Let $S(t)$ be the number of hours Fernando spent swimming in week t, and let $C(t)$ be the number of hours he spent taking apart computers. Assume $S(t)$ and $C(t)$ are continuous functions of time.

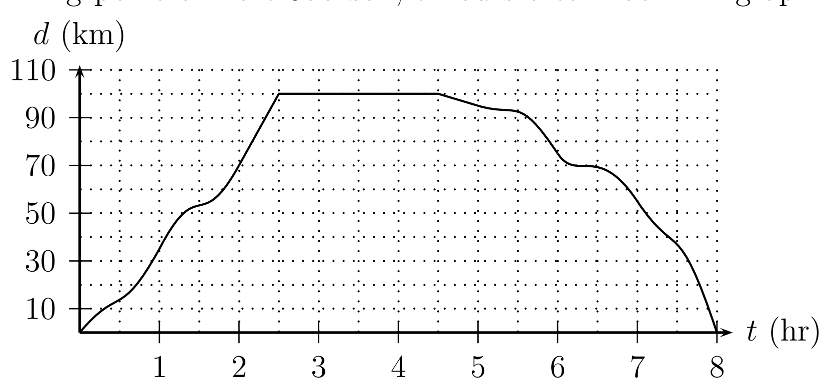


   (a) What does it mean for a function to be continuous?

   (b) Are $S(t) + C(t)$, $S(t) - C(t)$, and $S(t)C(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Fernando was spending the same amount of time swimming and taking apart computers.

9. (This problem appeared on a Winter, 2014 Math 115 Exam) A ship's captain is making a round trip voyage between two ports. The ship sets sail from Port Jackson at noon, arrives at Port Kembla some time later, waits there for a while, and then returns to Port Jackson. Let $s(t)$ be the ship's distance, in kilometers, from its starting point of Port Jackson, t hours after noon. A graph of $d = s(t)$ is shown below.



   (a) How far is Port Kembla from Port Jackson?

   (b) How long does the ship wait in Port Kembla?

   (c) What is the ship's average speed during the return trip?

   (d) Estimate the ship's instantaneous velocity at 1pm.

   (e) Sometime after 5pm, there is a time when the ship's instantaneous velocity is 0 km/hr. When does this occur?