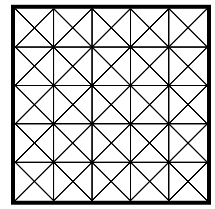
Douglass Houghton Workshop, Section 1, Wed 9/21/16

Worksheet Eat Cake (Let Them)



1. Cake! We had a number of ideas last time. One was to cut the cake up into lots of little triangles, and give each person the right number of them, making sure everyone gets the same amount of edge frosting.

   We want a general solution for n people, and especially solutions for 10 and 20 people.

2. We're still sitting trying to figure out the temperature in Fahrenheit, given a Celcius thermometer, a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, and a stopwatch.

3. Consider the double Ferris wheel: `http://www.youtube.com/watch?v=2DV4hN0c8WU`

   (a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

   (b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

   (c) Suppose Jannah is seated at one end of the big arm, i.e., at the center of one of the small wheels. Assume she starts as far to the right as possible. Write a formula for her height t seconds after the wheel starts, relative to the center of the big wheel.

   (d) Do the same for Jannah's horizontal position.

   (e) Now suppose Jannah's twin sister is in a seat on one of the small wheels. Write formulas for her x and y position *relative to Jannah.*

   (f) Now find formulas for Jannah's sister's position relative to the center of the big wheel.

4. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

5. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head and Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% annual interest, compounded annually.

   (b) 5% annual interest, compounded monthly.

   (c) 4% annual interest, compounded daily.

   (d) annual interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

6. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r.$$

   (b) Devise a similar rule for the amount of time it takes your money to triple at r% interest.

7. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?