Douglass Houghton Workshop, Section 1, Mon 9/19/16

Worksheet Dragon

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

(a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

(b) What's the formula for $N(T)$? (We found this last week).

(c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

(d) Verify your guess by finding a formula that fits the data.

(e) Using the formula we found last week for splitting the towel into n parts, write a limit equation to express the result in part (d).

2. Consider the double Ferris wheel: `http://www.youtube.com/watch?v=2DV4hN0c8WU`

(a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

(b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

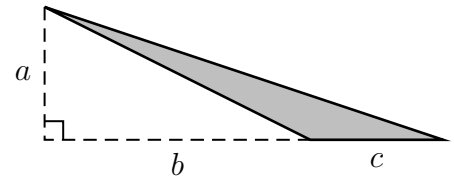
(c) Suppose Jannah is seated at one end of the big arm, i.e., at the center of one of the small wheels. Assume she starts as far to the right as possible. Write a formula for her height t seconds after the wheel starts, relative to the center of the big wheel.

(d) Do the same for Jannah's horizontal position.

(e) Now suppose Jannah's twin sister is in a seat on one of the small wheels. Write formulas for her x and y position *relative to Jannah.*

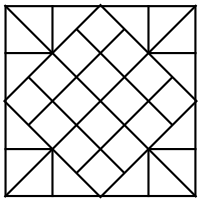
(f) Now find formulas for Jannah's sister's position relative to the center of the big wheel.

3. Find the area of the shaded triangle:



4. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides.

   It is a yellow cake with chocolate frosting. It's getting a bit drippy while we decide how to cut it. We need to be able to cut it for any number of people. Currently we can do 2, 4, 8, and 16. The solution for 16 involves separating the corners from the interior, and giving people one piece from each.

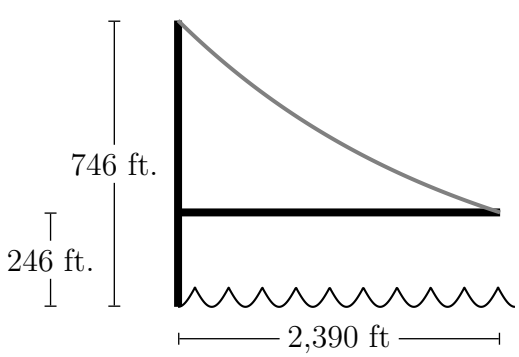


   It was suggested that we might generalize this idea by cutting a square out of the middle of the cake, and then allocating the rest fairly.

5. dBase™ was a database management system popular on IBM PCs back in the 1980s. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

6. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

7. (This question appeared on a Fall, 2008 Math 115 exam.) San Francisco's famous Golden Gate bridge has two towers which stand 746 ft. above the water, while the bridge itself is only 246 ft. above the water. The last leg of the bridge, which connects to Marin County, is 2,390 ft. long. The suspension cables connecting the top of the tower to the mainland can be modeled by an exponential function. Let $H(x)$ be the function describing the height above the water of the suspension cable as a function of x, the horizontal distance from the tower.



   (a) Find a formula for $H(x)$.

   (b) The engineers determined that some repairs are necessary to the suspension cables. They climb up the tower to 400 ft above the bridge, and they need to lay a horizontal walking board between the tower and the suspension cable. How long does the walking board need to be to reach the cable?