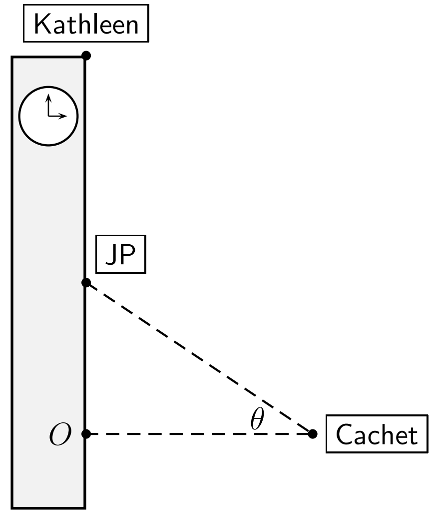
Douglass Houghton Workshop, Section 2, Tue 12/6/11

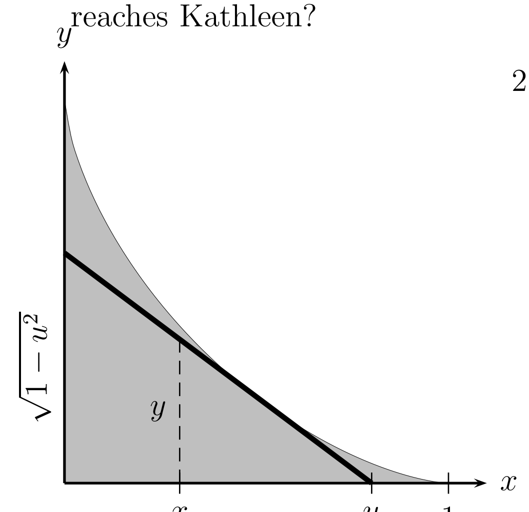
Worksheet Questions for the Ages



1. (Adapted from a Winter, 2005 Math 115 exam) As we know, Kathleen plays 4 instruments, including piano. One day when Cachet is working on a grounbreaking new dance style which demands a dramatic debut, she convinces Kathleen to climb to the top of Burton Tower and commandeer the carillon, so that Cachet can dance to the music. Since Kathleen locked the door on her way up, JP, as part of his baseball training regimen, is given the mission of climbing up the side of the tower to retake the bells. Assume Cachet is 30 ft from the base of the tower, and θ is the angle she has to look up to see JP.

   (a) Find a formula for the rate of change of JP's distance from the point O with respect to θ.

   (b) If the distance from point O to Kathleen is 200 ft. and JP is climbing at a constant 8 ft/sec, what is the rate of change of θ with respect to time when JP reaches Kathleen?



2. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time you found that if the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is

$$y = \frac{u - x}{u}\sqrt{1 - u^2}.$$

   (a) What value of u maximizes y? (Keep x fixed!)

   (b) So what is the maximum value of y? It's a function of x, of course.

   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

3. (This problem appeared on a Winter, 2004 Math 115 Final Exam.) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$, measured in songs/hour where $t = 0$ corresponds to 5 pm. Explain the meaning of the quantity $\int_0^5 m(t)\,dt$.

4. (This problem appeared on the same exam as above.) Let f be a continuous differentiable function of x. Suppose f is always increasing. The following is a table of values of $f(x)$.

x	.8	.9	1	1.1	1.2	1.3	1.4	1.5
$f(x)$	3	25	26	27	49	52	62	63

   (a) Using the table above, give an approximation of $f'(1)$.

   (b) Would a left-hand or a right-hand sum give a lower estimate of $\int_1^{1.5} f(x)\,dx$? Why?

   (c) Using the table above, give upper and lower estimates of $\int_1^{1.5} f(x)\,dx$.

5. (This problem appeared on the Winter, 2004 Math 115 Final Exam) It is estimated that the rate people will visit a new theme park is given as

$$r(t) = \frac{A}{1 + Be^{-0.5t}}$$

where A and B are both constants and $r(t)$ is measured in people per day, and $t = 0$ corresponds to opening day.

   (a) Write an integral that gives the total number of people visiting the park in the first year it is open. Do not try to evaluate the integral!

   (b) Suppose that $A = 100$ and $B = 5$. Given that

$$\frac{d}{dt}\left(2A\ln(1 + Be^{-0.5t}) - 2A\ln(Be^{-0.5t})\right) = \frac{A}{1 + Be^{-0.5t}},$$

   use the First Fundamental Theorem of Calculus to evaluate how many people visit the park during the first year it is open. Make sure you clearly indicate your use of the theorem.

6. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

7. Suppose $\int_a^b f(x)\,dx = 2$ and $\int_a^b g(x)\,dx = 4$. Evaluate the following expressions, if possible. Assume that all functions are continuous on the interval $[a, b]$.

   (a) $\int_a^b (g(x))^2\,dx - \left(\int_a^b g(x)\,dx\right)^2$

   (b) $\int_{a+2}^{b+2} f(x - 2)\,dx$

   (c) $\int_a^b (f(x)g(x))\,dx$

   (d) $\int_b^a (g(x))\,dx$