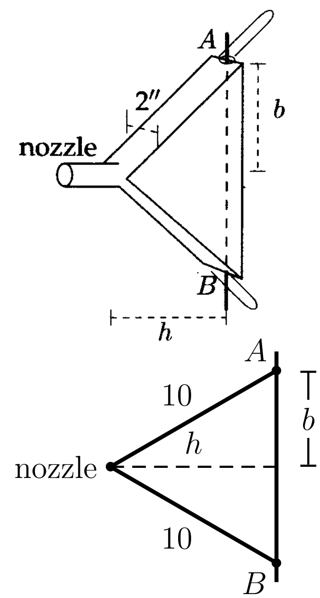
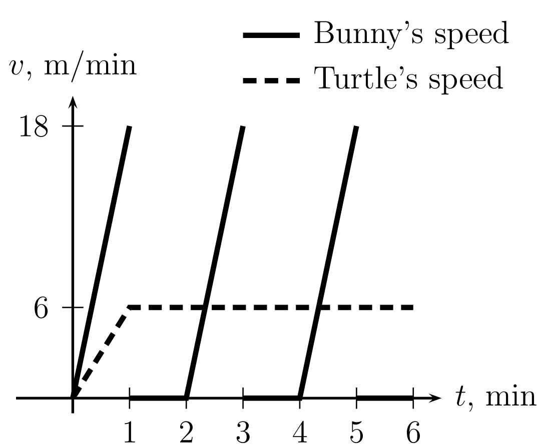
Douglass Houghton Workshop, Section 2, Thu 12/1/11

Worksheet Partridge

1. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



2. (This problem, like all Awkward Turtle problems, appeared on a Winter 2009 Math 115 Exam.) The Awkward Turtle is competing in a race! Unfortunately his archnemesis, the Playful Bunny, is also in the running. The two employ very different approaches: the Awkward Turtle takes the first minute to accelerate to a slow and steady pace which he maintains through the remainder of the race, while the Playful Bunny spends the first minute accelerating to faster and faster speeds until shes exhausted and has to stop and rest for a minute - and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)

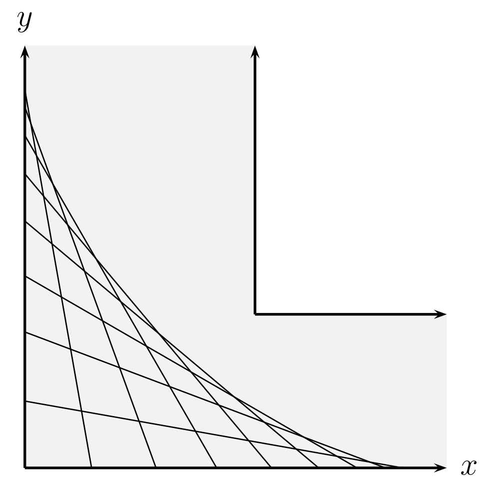


   (a) What is the Awkward Turtle's average speed over the first two minutes of the race? What is the Playful Bunny's?

   (b) The Playful Bunny immediately gets ahead of the Awkward Turtle at the start of the race. How many minutes into the race does the Awkward Turtle catch up to the Playful Bunny for the first time?

   (c) If the race is 60 meters total, who wins?

3. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

4. (Adapted from a fall, 2004 Math 115 final) Adam discovers that he *loves* eggnog, and one day he goes on an eggnog binge. The rate at which he drinks is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in liters/hour. Suppose $t = 0$ corresponds to 10 am.

   (a) Write a definite integral that represents the total amount of eggnog Adam consumes between noon and 10 pm.

   (b) If Adam's rate of drinking is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of eggnog he drinks in the first four hours of his binge.

   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

5. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

   Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

   (a) $\int_4^6 f(x-4)\,dx$

   (b) $\int_{-2}^0 2g(-t)\,dt$

   (c) $\int_2^0 (f(y)+2)\,dy$

   (d) $\int_2^2 g(x)\,dx$

   (e) Suppose that f is an even function. Find the average value of f from -2 to 2.