Worksheet Out Among the Stars

1. SHORTEST NETWORK. We found last week that some V newtorks can be improved to a Y, and some can't. In particular, our big result was that:

   If A, B, and C form an isosceles triangle, then the V network is not minimizing if the vertex angle is less than $120°$.

   Now we need to generalize to all possible placements of cities.

   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains _____ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.
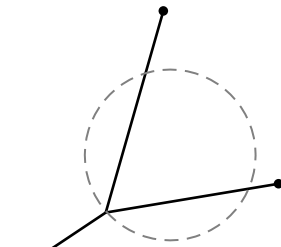
2. Write the following sums in sigma ($\sum$) notation.

   (a) $1 + 2 + 3 + 4 + \cdots 10$

   (b) $1 + 2 + 3 + 4 + \cdots + n$

   (c) $3 + 5 + 7 + 9 + \cdots + 21$

   (d) $4 + 9 + 16 + 25 + \cdots + 100$

   (e) $2.3 + 2.8 + 3.3 + 3.8 + 4.3 + 4.8 + \cdots + 10.3$

   (f) $f(a_1) + f(a_2) + f(a_3) + \cdots + f(a_n)$

3. (From the Winter, 2011 Math 115 Final) The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

   (a) Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

   (b) How frequently over the 54 week period would you need the data for $g(t)$ to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 0.5 lb (8 oz)?

4. Let $s(t)$ give the position of a truck moving along a straight highway at time t, and let $v(t)$ denote the truck's insantaneous velocity at time t.

   (a) What is the definition of the *average velocity* of the truck over the time interval from $t = a$ to $t = b$? (Think back a couple of months...)

   (b) What is the definition of the *average of the velocity function* over the interval from $t = a$ to $t = b$?

   (c) Are the quantities in (a) and (b) equal? Why or why not?

5. Consider the function $f(x) = x^x$.

   (a) It's neither a power function (ax^b) nor an exponential (ab^x). Nevertheless, find its derivative. Hint: rewrite it in the form $e^{u(x)}$ for some function u.

   (b) What is the minimum value that f takes on? (Check with your calculator, but find the answer with calculus.)

6. Solid angle is a measure of how much of your field of vision an object takes up. It's the area the object takes up when it is projected onto a sphere of radius 1 centered on your eye.

   The quality of your TV-watching experience is measured by the solid angle that the TV takes up. The solid angle of a $w \times h$ rectangle a distance r away is

   $$\Omega = 4\tan^{-1}\left(\frac{wh}{2r\sqrt{4r^2 + w^2 + h^2}}\right).$$

   (a) If the aspect ratio is 16×9 and we sit 200 inches away, that reduces to

   $$\Omega = 4\tan^{-1}\left(\frac{.0011\ell^2}{\sqrt{160000 + \ell^2}}\right)$$

   for a TV of diagonal length ℓ. Find and interpret $d\Omega/d\ell$ when $\ell = 60''$.

   (b) Here are some current prices for Samsung plasma TVs, from the Best Buy website:

Size	43″	51″	59″
Price	$500	$800	$1200

   Make up a model for the price p of a TV of size ℓ. (Don't make it linear—that's boring.)

   (c) Compute $d\Omega/dp$ when $\ell = 60''$. Interpret that in a sentence.

   (d) Suppose you're a restaurant owner, and you know that your revenue over the life of the TV will be $A + B\Omega$. If the price of the TV is our only cost, and a $60''$ TV is the size that makes the most profit, what is B?