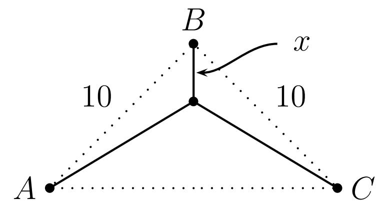
Douglass Houghton Workshop, Section 2, Tue 11/22/11

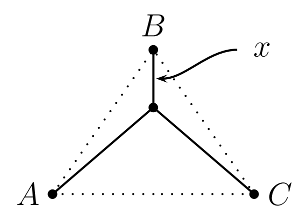
Worksheet Never Say Never

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 10 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 20. But by constructing a Y-shaped network like the one at the right, we found

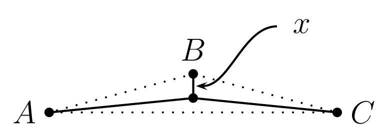
   

   - The length of the network is $x + 2\sqrt{100 - 20x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 20$) by increasing x. For instance, when $x = 5$, the network has length 19.74.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 10, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 20$)?

   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. Section 4.7 of your book is one that we will skip, but it covers a very important idea called L'Hôpital's Rule, which allows us to evaluate some limits we couldn't do otherwise. Look at the first box on page 229 for the simplest statement of it.

   We'd like to prove the formula for Michael Phelps's wetness that we found back in September. Recall that we reasoned he had

   $$L = \lim_{n\to\infty}\left(\frac{1}{1 + T/n}\right)^n$$

   liters of water on him after splitting the towel many times.

   (a) Take the ln of both sides of the equation above. It's OK to move the ln inside the limit. Why?

   (b) Let $p = \frac{1}{n}$, and change the equation so it's in terms of p. Manipulate until the thing inside the limit is 0/0 when evaluated.

   (c) Now apply L'Hôpital's Rule, and solve for L.

3. (Adapted from a Winter, 2006 Math 115 exam) Adam usually travels home to Menominee via Chicago, but tomorrow he decides to take the scenic route through the UP. As he drives north across the Mackinac Bridge, he sights his favorite fudge factory, located 1/4 mile due East of the end of the bridge. He might be fooling himself, but the minute he sights the factory he is sure he can smell the chocolate fudge. He has to put the car on cruise control (at 55 mph) to resist the temptation to speed the rest of the way across the bridge.

   (a) If Adam is 1.25 miles from the end of the bridge when he spots the factory, what is the distance (across the water) between his car and the factory?

   (b) If θ is the angle formed by a line between the factory and the end of the bridge and the line from the factory to Adam's car, how fast is θ changing at the time Adam spots the factory?

   (c) Because the wind is out of the east, the strength of the smell of the fudge, in units of "intoxicants", is the cosine of θ divided by the distance from the car to the factory. Find the rate at which the smell is becoming stronger when Adam sights the car.

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day in November she uploads her manuscript to a website (`http://www.nanowrimo.org/user/219891`), which counts how many words she has written. Here are her counts as of this afternoon, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	6	10000	11	18400	16	25500	21	35500
2	3300	7	11100	12	19700	17	27200	36500	
3	5000	8	13600	13	21000	18	29300		
4	7000	9	15000	14	21300	19	31400		
5	8300	10	16800	15	22700	20	33400		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. Draw a graph of $W(x)$ for x from 13 to 18.

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 13 to 18.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 13$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(13), F(14), \ldots, F(18)$. What do you notice? Explain this result.