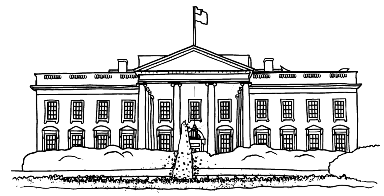
Douglass Houghton Workshop, Section 2, Tue 11/1/11

Worksheet Koala



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Near the surface of the earth, the force of gravity makes falling objects accelerate downward at the constant rate of $32\,\mathrm{ft/sec}^2$. What, therefore, is $v(t)$, the velocity of a falling object t seconds after it is dropped? Note $v'(t)$ is acceleration.

   (b) What, then, is $h(t)$, the height of an object t seconds after it is dropped from a height H? Note $h'(t)$ is velocity and $h(0) = H$.

   (c) How long does it take an object to fall from the top of the real White House, which is 70 ft high? So how many frames should we *show* for the fall?

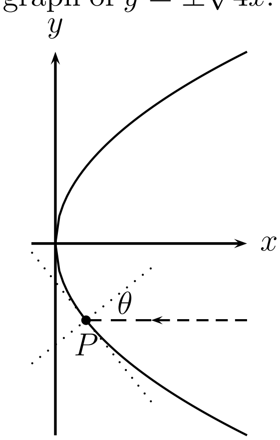
   (d) How long does it take an object to fall from the top of the model?

   (e) How many frames per second should you film at?

2. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   So far we've found:

   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle θ with the x-axis, it has slope $\tan\theta$.
   - So if we call the angle between the normal line and the horizontal θ, then $\tan\theta = b/2$.



   (a) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the x-axis?

   (b) What is the slope of the reflected ray? Put your answer in terms of b. Hint: $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$.

   (c) Write an equation for the reflected ray.

   (d) Where does the reflected ray intersect the x-axis? What is surprising about this answer?

   (e) Graph several rays, with their reflections.

   (f) What's cool about this type of mirror?

3. (This problem appeared on a Fall, 2006 Math 115 exam) The Flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F = kR^4$ for some positive constant k.

   (a) Find a linear approximation for F as a function of R near $R = 0.5$. (Leave your answer in terms of k).

   (b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1mm.

   (c) Is the answer you found in part (b) an under- or over-approximation? Justify your answer.

4. Molecules absorb far-infrared radiation because its excites their rotation. The absorption coefficient a of a given liquid varies with the frequency ω of the radiation according to

$$a(\omega) = \frac{10}{\omega^2 - 2c\omega + 125}$$

   where c is some constant ($0 \le c \le 11$).

   (a) For what value of the frequency ω is the absorption a maximum?

   (b) Graph $a(\omega)$ for $c = 11$. How would you describe the shape of this graph?

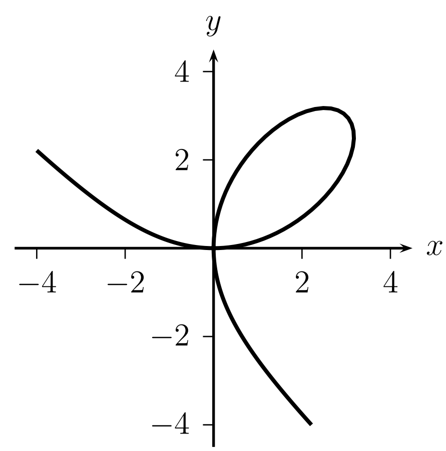
5. (This problem appeared on a Winter, 2005 Math 115 Exam) An example of Descartes' folium, shown in the picture to the right, is given by $x^3 + y^3 = 6xy$.

   (a) Show that the point $(3, 3)$ is on the graph.

   (b) Find the equation of the tangent to the graph at the point $(3, 3)$.

   (c) For what value(s) of x will the tangent to this curve be horizontal? [You do not need to solve for both x and y—just show x in terms of y.]

   (d) (Added for DHSP) Oh heck, go ahead and find the point(s).



6. Let $f^{(n)}(x)$ denote the nth derivative of f. If $f(x) = e^{-2x}$, find $f^{(531)}(x)$. Is $f^{(531)}(x)$ increasing or decreasing? Concave up or concave down? Try graphing $f^{(531)}$ without your calculator, then check with the calculator.