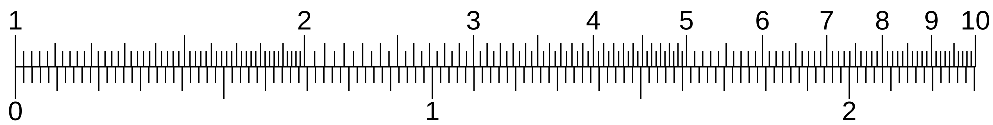
Douglass Houghton Workshop, Section 2, Thu 9/29/11

Worksheet Harbinger of Things to Come

1. Explain how you might use two long rulers to add numbers mechanically. (Example: find $2 + 3$ with two 6-inch rulers.)

2. Explain how a slide rule is able to multiply numbers. It might help to recall that this picture:

   represents the function $\ln(x)$ if we find x on the top scale and read downward. Note that the bottom scale is just like a regular ruler, in some units.

3. (This problem appeared on a Fall, 2008 Math 115 Exam) The speed of sound, $v(T)$ (in miles per hour), at an ambient temperature, T (in degrees Fahrenheit), is given by:

   $$v(T) = 740 + 0.4T.$$

   Objects which travel faster than the speed of sound create *sonic booms*. However, the ambient temperature T in the Troposphere also decreases with height h (in miles) from the earth's surface according to the equation

   $$T(h) = -26h + T_0,$$

   where T_0 is the temperature at the surface.

   (a) Find a formula which will give the speed of sound S as a function of height h, assuming the surface temperature is $68°$.

   (b) Find $S'(1)$ and interpret the meaning of $S'(1)$ in the context of this problem.

   (c) While on a flight from Ann Arbor to Chicago on a beautiful $68°$ day, the pilot's instruments measure the outside temperature to be $0°$. What is the plane's altitude, and how fast would the pilot need to fly at this altitude to create a sonic boom?

4. (This problem appeared on a Winter, 2009 Math 115 Exam) Air pressure, P, decreases exponentially with the height, h, in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

   (a) On top of Mount McKinley, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

   (b) Determine $P^{-1}(0.7)$. Include units!

5. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

6. (This problem appeared on a Winter, 2009 Math 115 Exam) A continuous function f, defined for all x, is always decreasing and concave up. Suppose $f(6) = -6$ and $f'(6) = -1.5$.

   (a) How many zeroes does f have? Justify your answer.

   (b) Can $f'(2) = -1$? Justify your answer.

   (c) Circle all the intervals below in which f has at least one zero. Justify your choices with a picture and a short description.

$$(-\infty, -6) \quad [-6, -2) \quad [-2, -1) \quad [-1, 1) \quad [1, 2) \quad [2, 6) \quad [6, \infty)$$

7. (This problem appeared on a Winter, 2007 Math 115 Exam) Cosmologists, through a technique best described as hocus pocus, measure a quantity $T(t)$, the temperature of the universe in degrees Kelvin (K), where t is in gigayears (Gyr) after the Big Bang. Suppose that, currently, $t = 13.6$, $T(13.6) = 2.4$, and $T'(13.6) = -12$.

   [Note: A gigayear is 1 billion years, and the Kelvin temperature scale is an absolute temperature scale where the lowest possible temperature is defined as being zero Kelvin.]

   (a) For each of the following statements, state whether you agree or disagree with the conclusion and justify your reasoning.

      i. In the next billion years, the temperature of the universe will drop by approximately 12 degrees Kelvin.

      ii. In the next year, the temperature of the universe will drop by approximately $\frac{12}{1{,}000{,}000{,}000}$ degrees Kelvin.

   (b) Assume $T(t)$ is decreasing and does not change concavity on the domain $[13.6, \infty)$. Do you expect $T(t)$ to be concave up or concave down on the domain $[13.6, \infty)$? Justify your answer using physical reasoning.

8. (This problem appeared on a Winter, 2011 Math 115 exam.) Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

x	0	0.5	1
$f(x)$	0	0.247	0.841

   (a) Estimate $f'(1)$. Be sure it is clear how you obtained your answer.

   (b) Estimate $f''(1)$. Again, be sure it is clear how you obtained your answer.

   (c) Estimate $f(1.25)$, being sure your work is clear.

   (d) Based on your work in (a) and (b), is your estimate in (c) an over- or under-estimate? Explain.