Douglass Houghton Workshop, Section 2, Thu 9/22/11

Worksheet Fluffernutter

1. Last time we investigated a rule for how a population of fish might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	

An *equilibrium* is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is *stable* if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is *unstable*.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words. Note: these may be harder to explain in terms of fish, but it will be fun to try.

$$P(n+1) = .75P(n) + 200$$
$$P(n+1) = .4P(n) + 600$$
$$P(n+1) = 1.1P(n) - 330$$
$$P(n+1) = -.5P(n) + 1200$$
$$P(n+1) = -1.3P(n) + 460$$
$$P(n+1) = P(n) + 300$$
$$P(n+1) = -P(n) + 300$$

(b) Explain how to find the equilibrium and its stability for the general linear recurrence

$$P(n+1) = mP(n) + b.$$

where m and b are constants.

2. Let $f_0(x) = 2x^3 - 3x^2 + 7x - 1$.

(a) Let $c_0 = \lim_{x\to 0} f_0(x)$. Find c_0.

(b) Let $f_1(x) = \frac{f_0(x) - c_0}{x}$, and let $c_1 = \lim_{x\to 0} f_1(x)$. Find c_1.

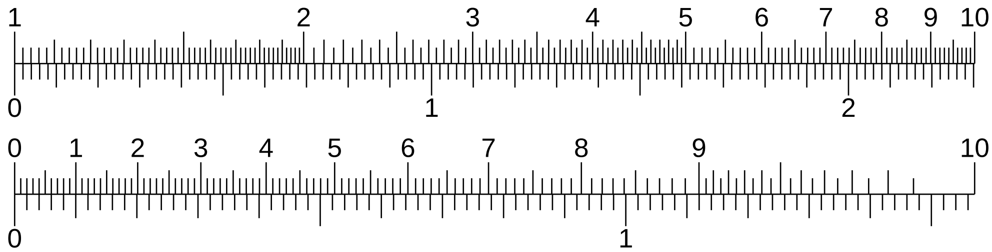
(c) Let $f_2(x) = \frac{f_1(x) - c_1}{x}$, and let $c_2 = \lim_{x\to 0} f_2(x)$. Find c_2.

(d) Likewise find c_3. What about c_4, c_5, etc.?

(e) Find all the c's when $f_0(x) = 3x^2 - 2x - 5$.

(f) Given a polynomial $f_0(x)$, can you see how to get the c's without a lot of effort? Try to explain why your method works.

3. What's the deal with these pictures? What are they good for?



4. (This problem appeared on a Fall, 2005 Math 115 Exam) Suppose

$$f(x) = \begin{cases} e^{\sin(x)} & \text{if } x < \frac{\pi}{2} \\ kx & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

   (a) If f is continuous, what is the value of k?

   (b) Compute the average rate of change of f between $x = 1.5$ and $x = \frac{\pi}{2}$.

   (c) Compute the average rate of change of f between $x = 1.57$ and $x = \frac{\pi}{2}$.

   (d) Do you think f is differentiable at $x = \frac{\pi}{2}$?

5. Kyle has noticed that his tastes have changed some since last year. Back then he spent about 15 hours a week playing hockey, and 10 hours practicing Taekwando. Gradually school took over his life, and though there have been some ups and downs in his schedule, the general trend is that he's spent less time per week on both. Now, 52 weeks later, he spends only 3 hours a week playing hockey and 5 hours a week on Taekwondo.

   Let $H(t)$ be the number of hours Kyle spent playing hockey in week t, and let $T(t)$ be the number of hours he spent kicking people in the dojo. Assume $H(t)$ and $T(t)$ are continuous functions of time.

   (a) Are $H(t) + T(t)$, $H(t) - T(t)$, and $H(t)T(t)$ continuous?

   (b) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Kyle was spending the same amount of time on hockey and Taekwondo.

6. Last time we found a way to construct a fair die for any even number of sides. Prove that it's possible to construct a fair 5-sided die. Some conditions:

   - All 5 sides must be flat.
   - No handles (ala a dradle) allowed.