Douglass Houghton Workshop, Section 2, Tue 9/20/11

Worksheet Emphatic

1. Examine the YouTube video of the double Ferris wheel:

   `http://www.youtube.com/watch?v=xj6DVY5s8HU`

   Assume that when the wheel starts the big arm is horizontal, and you are seated in a chair which is as far to the right as a chair can get.

   (a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

   (b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

   (c) Write a formula for your height t seconds after the wheel starts.

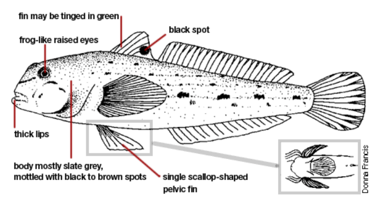
   (d) Do the same for your horizontal position.

   (e) Draw a two-dimensional picture of your motion, and mark some times on the picture. Then watch the video again and see if it looks right.

2. Taylor is studying the population of Round Gobies, an invasive fish species, in Lake Michigan. Suppose that the population changes according to the rule:

   $$P(n+1) = 1.5P(n) - 200$$

   where $P(0)$ is the population in 2011, $P(1)$ is the population 1 year later, etc.

   (a) Make up a (short) story about Round Gobies that yields that formula as the result.

   (b) Suppose there are 320 fish in 2011. What will happen in the long run?

   (c) Suppose instead that there are 800 fish in 2011. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

3. Repeat the last problem, but for the rule

   $$P(n+1) = .75P(n) + 200.$$

4. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two fish scenarios has a stable equilibrium?

5. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) from his Frosted Flakes™ endorsement deal, before Kellogg's dropped him, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

6. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you in vest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r.$$

   (b) Devise a similar rule for the amount of time it takes your money to triple at r% interest.

7. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?