Douglass Houghton Workshop, Section 2, Thu 9/15/11

Worksheet Dauntless

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

   (a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

   (b) What's the formula for $N(T)$? (We found this a couple of days ago).

   (c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

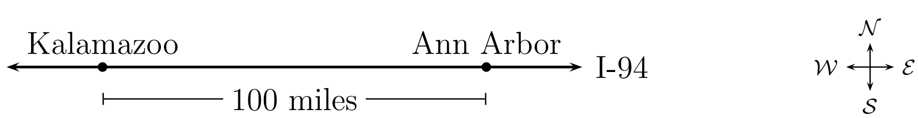
   (d) Verify your guess by finding a formula that fits the data.

   (e) Try to explain the relationship between $M(1)$ and $M(2)$.

   (f) Using the formula we found on Tuesday, write a limit equation to express the result in part (d).

2. Kalamazoo is 100 miles west of Ann Arbor along Route 94. Suppose we know the temperature at every point on the road between the two cities, and we express that information as

$$T(x) = \text{ the temperature in Fahrenheit at a point } x \text{ miles west of Ann Arbor.}$$



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

3. dBase™ was a database management system popular on IBM PCs back in the 80s. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

4. Examine the YouTube video of the double Ferris wheel:

   `http://www.youtube.com/watch?v=xj6DVY5s8HU`

   Assume that when the wheel starts the big arm is horizontal, and you are seated in a chair which is as far to the right as a chair can get.

   (a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

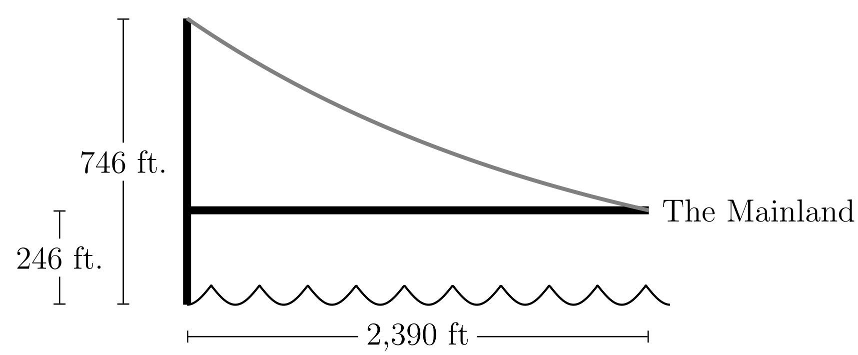
   (b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

   (c) Write a formula for your height t seconds after the wheel starts.

   (d) Do the same for your horizontal position.

   (e) Draw a two-dimensional picture of your motion, and mark some times on the picture. Then watch the video again and see if it looks right.

5. (This question appeared on a Fall, 2008 Math 115 exam.) San Francisco's famous Golden Gate bridge has two towers which stand 746 ft. above the water, while the bridge itself is only 246 ft. above the water. The last leg of the bridge, which connects to Marin County, is 2,390 ft. long. The suspension cables connecting the top of the tower to the mainland can be modeled by an exponential function. Let $H(x)$ be the function describing the height above the water of the suspension cable as a function of x, the horizontal distance from the tower.



   (a) Find a formula for $H(x)$.

   (b) The engineers determined that some repairs are necessary to the suspension cables. They climb up the tower to 400 ft above the bridge, and they need to lay a horizontal walking board between the tower and the suspension cable. How long does the walking board need to be to reach the cable?