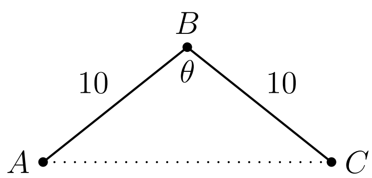
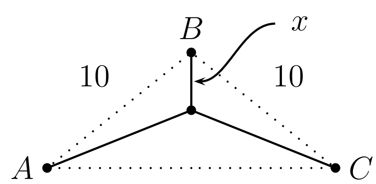
Douglass Houghton Workshop, Section 1, Mon 11/28/11

Worksheet Omnibus

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one at the right. We know:

   - When you have a Y-shaped network like the one shown below, its total length is
   $$L_\theta(x) = x + 2\sqrt{x^2 - 20x\cos(\theta/2) + 100}.$$
   - For $x = 0$, that's just the V-shaped network with roads from A to B and B to C.
   - When the vertex angle is 90°, then sometimes the Y beats the V. For instance,
   $$L_{90}(3) \approx 19.3 < 20 = L_{90}(0)$$
   so placing the roundabout 3 miles south of B is better than not having a roundabout at all.





   (a) What about $\theta = 150°$? Try to find an x which improves the V.

   (b) Put your calculator in degrees mode and plot $L_{90}(x)$ and $L_{150}(x)$, for x from 0 to 10. The shapes are about the same, but what's the difference that explains the fact that you can improve some V-shaped networks and not others? (Remember the V is $x = 0$.)

   (c) So which V's can be improved, and which can't? State the result in the form: "Any V-shaped network with an angle smaller than ______ can be improved".

2. Write the following sums in sigma ($\sum$) notation.

   (a) $1 + 2 + 3 + 4 + \cdots 10$

   (b) $1 + 2 + 3 + 4 + \cdots + n$

   (c) $3 + 5 + 7 + 9 + \cdots + 21$

   (d) $4 + 9 + 16 + 25 + \cdots + 100$

   (e) $2.3 + 2.8 + 3.3 + 3.8 + 4.3 + 4.8 + \cdots + 10.3$

   (f) $f(a_1) + f(a_2) + f(a_3) + \cdots + f(a_n)$

3. Consider the function $f(x) = x^x$.

   (a) It's neither a power function (ax^b) nor an exponential (ab^x). Nevertheless, find its derivative. Hint: rewrite it in the form $e^{u(x)}$ for some function u.

   (b) What is the minimum value that f takes on? (Check with your calculator, but find the answer with calculus.)

4. Solid angle is a measure of how much of your field of vision an object takes up. It's the area the object takes up when it is projected onto a sphere of radius 1 centered on your eye.

   The quality of your TV-watching experience is measured by the solid angle that the TV takes up. The solid angle of a $w \times h$ rectangle a distance r away is

$$\Omega = 4\tan^{-1}\left(\frac{wh}{2r\sqrt{4r^2+w^2+h^2}}\right).$$

   (a) If the aspect ratio is 16×9 and we sit 200 inches away, that reduces to

$$\Omega = 4\tan^{-1}\left(\frac{.0011\ell^2}{\sqrt{160000+\ell^2}}\right)$$

   for a TV of diagonal length ℓ. Find and interpret $d\Omega/d\ell$ when $\ell = 60''$.

   (b) Here are some current prices for Samsung plasma TVs, from the Best Buy website:

Size	43″	51″	59″
Price	\$500	\$800	\$1200

   Make up a model for the price p of a TV of size ℓ. (Don't make it linear—that's boring.)

   (c) Compute $d\Omega/dp$ when $\ell = 60''$. Interpret that in a sentence.

   (d) Suppose you're a restaurant owner, and you know that your revenue over the life of the TV will be $A + B\Omega$. If a 60″ TV is the size that makes the most profit, what is B?

5. Let $s(t)$ give the position of a truck moving along a straight highway at time t, and let $v(t)$ denote the truck's insantaneous velocity at time t.

   (a) What is the definition of the *average velocity* of the truck over the time interval from $t = a$ to $t = b$? (Think back a couple of months...)

   (b) What is the definition of the *average of the velocity function* over the interval from $t = a$ to $t = b$?

   (c) Are the quantities in (a) and (b) equal? Why or why not?

6. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

   Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.