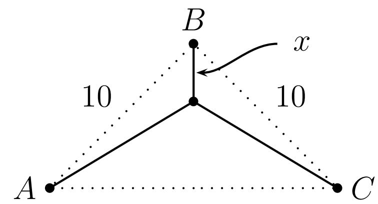
Douglass Houghton Workshop, Section 1, Mon 11/21/11

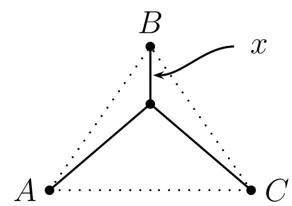
Worksheet Nigh on Thanksgiving

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 10 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 20. But by constructing a Y-shaped network like the one at the right, we found

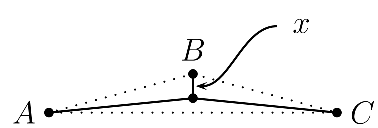
   

   - The length of the network is $x + 2\sqrt{100 - 20x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 20$) by increasing x. For instance, when $x = 5$, the network has length 19.74.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 10, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 20$)?

   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. Section 4.7 of your book is one that we will skip, but it covers a very important idea called L'Hôpital's Rule, which allows us to evaluate some limits we couldn't do otherwise. Look at the first box on page 229 for the simplest statement of it.

   We'd like to prove the formula for Michael Phelps's wetness that we found back in September. Recall that we reasoned he had

   $$L = \lim_{n\to\infty} \left(\frac{1}{1 + T/n}\right)^n$$

   liters of water on him after splitting the towel many times.

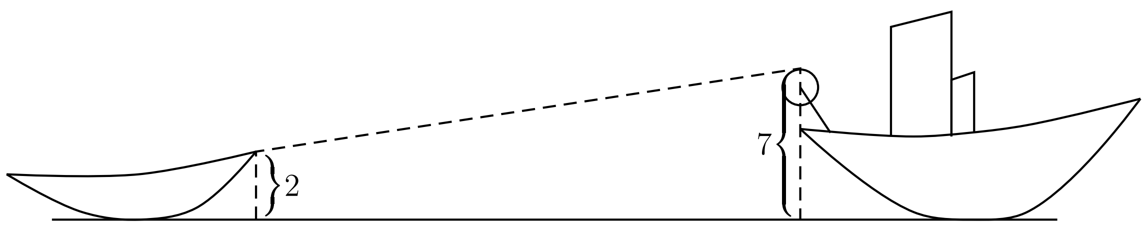
   (a) Take the ln of both sides of the equation above. It's OK to move the ln inside the limit. Why?

   (b) Let $p = \frac{1}{n}$, and change the equation so it's in terms of p. Manipulate until the thing inside the limit is 0/0 when evaluated.

   (c) Now apply L'Hôpital's Rule, and solve for L.

3. (Adapted from a Fall, 2006 Math 115 Final Exam.) By virtue of winning a Tae Kwon Do competition this year, Keon has been given the honor of dressing up as "Tae Kwon Tom", the Korean patron of Thanksgiving, who travels around the town delivering turkeys. At the moment, he is sailing the Detroit riverfront in a small boat. Unfortunately, he broke a few too many boards during the competition, which, together with his natural narcolepsy, caused him to fall asleep at the moment when he should have been filling the boat with gas.

   As a result, his boat needs to be towed. A cable is attached to the front of the boat 2 meters above the water. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of a tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



   (a) At what rate is the length of the cable between the two boats changing?

   (b) How fast is the small boat being pulled forward when it is 10 meters away from the tugboat?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day in November she uploads her manuscript to a website (`http://www.nanowrimo.org/user/219891`), which counts how many words she has written. Here are her counts as of this afternoon, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	6	10000	11	18400	16	25500	21	35100
2	3300	7	11100	12	19700	17	27200		
3	5000	8	13600	13	21000	18	29300		
4	7000	9	15000	14	21300	19	31400		
5	8300	10	16800	15	22700	20	33400		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. Draw a graph of $W(x)$ for x from 13 to 18.

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 13 to 18.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 13$, the line $x = t$, the y-axis, and the graph of $w(x)$. Make a table of values showing $F(13), F(14), \ldots, F(18)$. What do you notice? Explain this result.