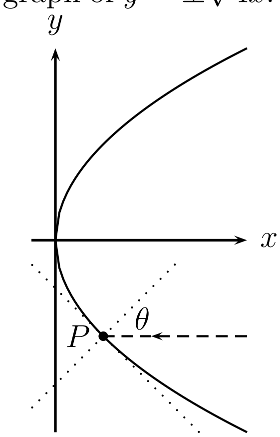
**Douglass Houghton Workshop, Section 1, Wed 11/2/11**

# Worksheet Labradoodle

1. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   So far we've found:

   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle $\theta$ with the $x$-axis, it has slope $\tan\theta$.
   - So if we call the angle between the normal line and the horizontal $\theta$, then $\tan\theta = b/2$.




   (a) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the $x$-axis?

   (b) What is the slope of the reflected ray? Put your answer in terms of $b$. Hint: $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$.

   (c) Write an equation for the reflected ray.

   (d) Where does the reflected ray intersect the $x$-axis? What is surprising about this answer?

   (e) Graph several rays, with their reflections.

   (f) What's cool about this type of mirror?

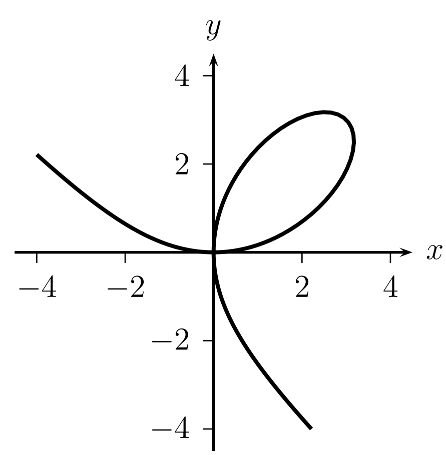
2. (This problem appeared on a Winter, 2005 Math 115 Exam) An example of Descartes' folium, shown in the picture to the right, is given by $x^3 + y^3 = 6xy$.

   (a) Show that the point $(3, 3)$ is on the graph.

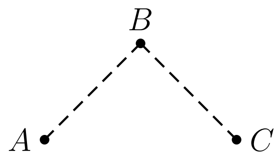
   (b) Find the equation of the tangent to the graph at the point $(3, 3)$.

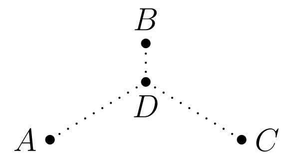
   (c) For what value(s) of $x$ will the tangent to this curve be horizontal? [You do not need to solve for both $x$ and $y$—just show $x$ in terms of $y$.]

   (d) (Added for DHSP) Oh heck, go ahead and find the point(s).



3. The three cities in the pictures below are at the corners of an 45°-45°-90° triangle whose legs are 10 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.




(Say, $A$ is Ann Arbor, $B$ is Flint, and $C$ is Port Huron.) The first, simple proposal (on the left) is to build a road from $A$ to $B$ and another from $B$ to $C$. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from $A$ and $C$ to a point $D$ just south of $B$, then building a road north from there to $B$.

(a) Let $x$ be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

(b) Calculate the total length of the new network in terms of $x$. Hint: "Law of cosines".

(c) Can you find a value of $x$ which will produce a shorter network than the simple proposal?

4. Let $f^{(n)}(x)$ denote the $n$th derivative of $f$. If $f(x) = e^{-2x}$, find $f^{(531)}(x)$. Is $f^{(531)}(x)$ increasing or decreasing? Concave up or concave down? Try graphing $f^{(531)}$ without your calculator, then check with the calculator.

5. (This problem appeared on a Fall, 2008 Math 115 exam) Determine $a$ and $b$ for the function of the form $y = f(t) = at^2 + b/t$, with a local minimum at $(1, 12)$.

6. Section 3.8 of your book (which we skip in 115) is about the "hyperbolic trig functions":

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

They are often called the even and odd parts of $e^x$, because they sum to $e^x$ and one is an even function and one is an odd function.

(a) Which is which?

(b) Let $f(x)$ be any old function which is defined for all real numbers $x$. Think of a way to split $f(x)$ into even and odd parts. (Hint: Stare at the definitions above until you get an idea. Then check it.)

(c) cosh and sinh obey many rules similar, but not exactly the same, as those for cos and sin. To deduce a few, find the derivatives of $\cosh(x)$ and $\sinh(x)$. Then find $\cosh(2x)$ and $\sinh(2x)$. Can you find something resembling $\sin^2 x + \cos^2 x = 1$?