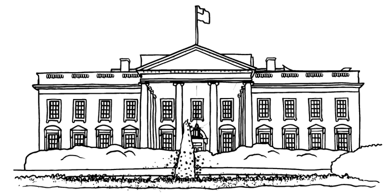
Douglass Houghton Workshop, Section 1, Wed 10/26/11

Worksheet Kangaroo



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Near the surface of the earth, the force of gravity makes falling objects accelerate downward at the constant rate of $32\,\text{ft}/\text{sec}^2$. What, therefore, is $v(t)$, the velocity of a falling object t seconds after it is dropped? Note $v'(t)$ is acceleration.

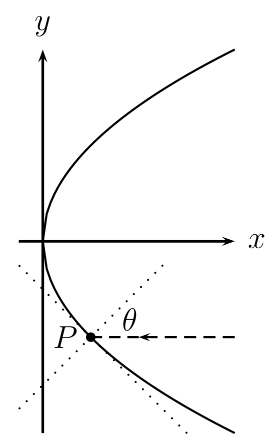
   (b) What, then, is $h(t)$, the height of an object t seconds after it is dropped from a height H? Note $h'(t)$ is velocity and $h(0) = H$.

   (c) How long does it take an object to fall from the top of the real White House, which is 70 ft high? So how many frames should we *show* for the fall?

   (d) How long does it take an object to fall from the top of the model?

   (e) How many frames per second should you film at?

2. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.



   (a) Draw the mirror on the board (make it big).

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). (Don't make up a number for b, just call it b.) At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (2d) and (2e).

   To be continued...

3. (This problem appeared on a Fall, 2006 Math 115 exam) The Flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F = kR^4$ for some positive constant k.

(a) Find a linear approximation for F as a function of R near $R = 0.5$. (Leave your answer in terms of k).

(b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1mm.

(c) Is the answer you found in part (b) an under- or over-approximation? Justify your answer.

4. Molecules absorb far-infrared radiation because its excites their rotation. The absorption coefficient a of a given liquid varies with the frequency ω of the radiation according to

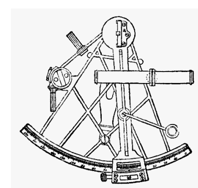
$$a(\omega) = \frac{10}{\omega^2 - 2c\omega + 125}$$

where c is some constant ($0 \leq c \leq 11$).

(a) For what value of the frequency ω is the absorption a maximum?

(b) Graph $a(\omega)$ for $c = 11$. How would you describe the shape of this graph?

5. Recall that one day while wakeboarding, Kelsey was watching her brother diving off a high platform. She happened to be carrying a sextant, which is a device for measuring angles. When the boat pulled her to a position 100 ft away from the platform, she measured the angle to the top to be θ.



If the measurement were exactly correct, the height of the tower would be $h(\theta) = 100\tan(\theta)$. But there is some small uncertainty about the sextant; all we know for sure is that the true angle is between $\theta - \epsilon$ and $\theta + \epsilon$.

(a) Use a derivative to estimate the uncertainty in the height, in terms of θ and ϵ.

(b) Now suppose that Sierra is on top of the tower, and she drops a tennis ball and times how long it takes to hit the ground. It's a ball signed by her favorite tennis player, Raphael Nadal. She uses the work from problem 1 to estimate the height. If her stopwatch is accurate to within a time δ, about how accurate is her estimate of the height?

(c) Suppose that $\epsilon = 0.01$ radians, $\delta = 0.1$ sec, and two measurements are $\theta = 0.29$ radians and falling time $= 1.4$ sec. What are the two estimates, and which is more accurate?