Douglass Houghton Workshop, Section 1, Mon 10/24/11

Worksheet Jumpin' Jehosaphat

1. Suppose you construct a $1/z$ scale model of the White House, in order to film it blowing up. You will show the film at 24 frames per second. How many frames per second should you *film* so that when you slow the speed down, things will fall at believable speeds?

   Hints:

   - A falling object has a constant acceleration. What does that mean about its velocity at time t?
   - What does *that* mean about its height at time t?
   - So how long would it take an object to fall off the top of the White House (say, 50 ft high)?

2. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

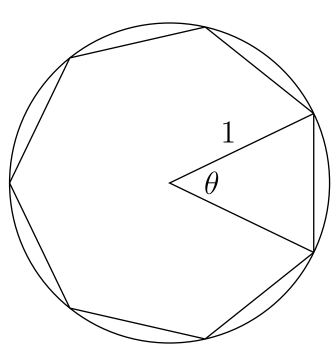
   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (2d) and (2e).

   To be continued...

3. (This problem explains why $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, but only when θ is measured in radians.) Consider a regular n-sided polygon inscribed in a circle of radius 1.

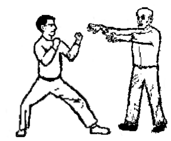


   (a) Last time we found that if we cut the n-sided polygon to the left into n triangles, then we can compute the area of each, and hence the area of the polygon, in terms of θ. Do it.

   (b) We also showed that $\theta = 2\pi/n$, so as $n \to \infty$, $\theta \to 0$. If A_n is the area of the polygon, then A_n approaches π as $n \to \infty$ (or as $\theta \to 0$), because the polygon approaches a circle. Use that to finish the proof.

4. There are at least three types of people: Humans, Zombies, and Dead Zombies. If there are some zombies and some humans, then when they meet some zombies will bite humans and turn them into zombies, and some humans will kill zombies and turn them into dead zombies. Also, sometimes zombies just die from lack of brains. Our job is to model the epidemic. Let

$$H(t) = \text{The number of humans at the end of day } t$$
$$Z(t) = \text{The number of zombies at the end of day } t$$
$$D(t) = \text{The number of dead zombies at the end of day } t$$



   (a) What are the changes in the number of humans from day t to day $t+1$? Write it like this:

   $$H(t+1) = H(t) + \boxed{}$$

   The change can depend on $H(t)$, $Z(t)$, and $D(t)$.

   (b) Likewise:

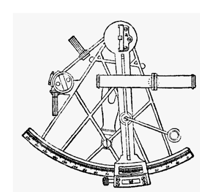
   $$Z(t+1) = Z(t) + \boxed{}$$
   $$D(t+1) = D(t) + \boxed{}$$

5. Let $f(x) = x^2 - 2x + 4$ and $g(x) = -x^2 - 2x - 3$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.

6. One day while wakeboarding, Kelsey watches her brother diving off a high platform. She wonders how high the platform is. As the boat pulls her past the platform, at a distance of 100 ft, she measures the angle to the top using a device called a sextant. The sextant has some inherent uncertainty; so let's say that Kelsey measures an angle θ, with an uncertainty of ϵ. So she knows that the real angle is somewhere between $\theta - \epsilon$ and $\theta + \epsilon$.



   (a) If the angle is exactly θ, how high is the platform?

   (b) Approximately how much uncertainty in that answer results from using the sextant? (Assume ϵ is small.)

   (c) How could Kelsey improve the accuracy of her measurement?