Douglass Houghton Workshop, Section 1, Wed 9/28/11

Worksheet Guacamole

1. Recall what this picture means:

   Then explain how a slide rule is able to multiply numbers.

2. Kelsey was a distance runner and also a sprinter on her high school track team. Her idol was Jamaican sprinter Usain Bolt, who, at the 2008 Beijing Olympics, shattered the old 100m world record with a new time of 9.69 seconds. Here were his split times:

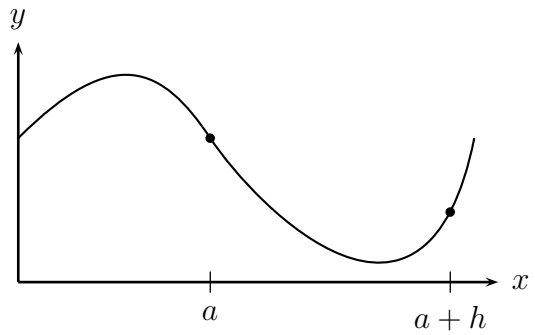
Distance (m)	0	10	20	30	40	50	60	70	80	90	100
Time (sec)	0	1.85	2.87	3.78	4.65	5.50	6.32	7.14	7.96	8.79	9.69

   Bolt did some celebrating at the end of the race, and therefore didn't run it as fast as he could have. Starting from the 60 meter mark, how would you guess that he should have finished? Try to fill in the rest of the table as if he hadn't celebrated.

3. (This problem appeared on a Fall, 2004 Math 115 exam) For this problem, f is differentiable everywhere.

   (a) Write the limit definition of the derivative of the function f at the point a.

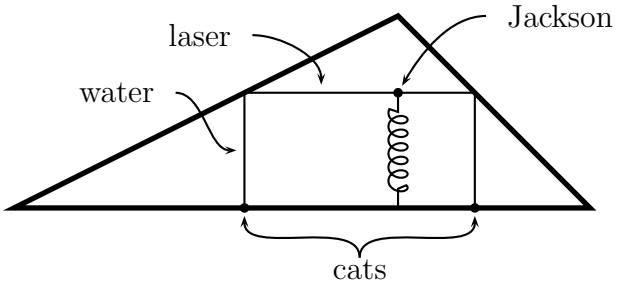
   (b) On the graph below, show how the rate of change of f between $x = a$ and $x = a + h$ is related to the derivative at the point a. Give a brief explanation of your illustration including how the limit as $h \to 0$ is demonstrated in your picture.



   (c) Write the limit definition for $f'(2)$ if $f(x) = e^{\sin 2x}$. [You do not need to find the limit or approximate $f'(2)$.]

   (d) (Added for DHSP) Oh, heck, go ahead and approximate $f'(2)$. Use your calculator, but not any fancy calculus features that it might have.

4. Jackson is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Jackson goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Jackson's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Jackson's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Jackson is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Jackson's height off the floor.

5. Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.

6. We considered several rules for how a population of fish might change. They were:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	no
$P(n+1) = .75P(n) + 200$		
$P(n+1) = .4P(n) + 600$		
$P(n+1) = -.5P(n) + 1200$		
$P(n+1) = -1.3P(n) + 460$		

   Find the stability of each.