Douglass Houghton Workshop, Section 1, Mon 9/26/11

Worksheet Fifty Million Frenchmen

1. Last time we investigated a rule for how a population of fish might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	

An *equilibrium* is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is *stable* if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is *unstable*.

   (a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words. Note: these may be harder to explain in terms of fish, but it will be fun to try.

$$P(n+1) = .75P(n) + 200$$
$$P(n+1) = .4P(n) + 600$$
$$P(n+1) = 1.1P(n) - 330$$
$$P(n+1) = -.5P(n) + 1200$$
$$P(n+1) = -1.3P(n) + 460$$
$$P(n+1) = P(n) + 300$$
$$P(n+1) = -P(n) + 300$$

   (b) Explain how to find the equilibrium and its stability for the general linear recurrence
$$P(n+1) = mP(n) + b.$$
   where m and b are constants.

2. Let $f_0(x) = 2x^3 - 3x^2 + 7x - 1$.

   (a) Let $c_0 = \lim_{x\to 0} f_0(x)$. Find c_0.

   (b) Let $f_1(x) = \frac{f_0(x) - c_0}{x}$, and let $c_1 = \lim_{x\to 0} f_1(x)$. Find c_1.

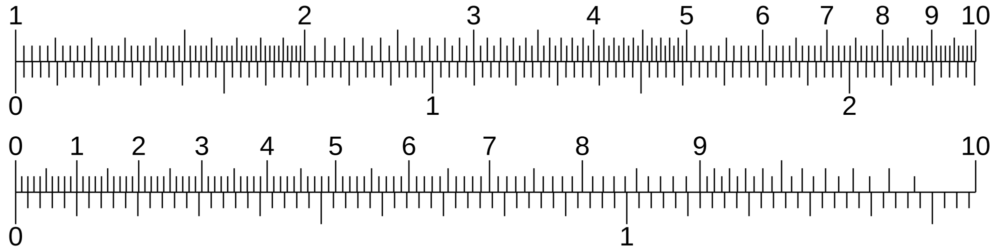
   (c) Let $f_2(x) = \frac{f_1(x) - c_1}{x}$, and let $c_2 = \lim_{x\to 0} f_2(x)$. Find c_2.

   (d) Likewise find c_3. What about c_4, c_5, etc.?

   (e) Find all the c's when $f_0(x) = 3x^2 - 2x - 5$.

   (f) Given a polynomial $f_0(x)$, can you see how to get the c's without a lot of effort? Try to explain why your method works.

3. What's the deal with these pictures? What are they good for?



4. (This problem appeared on a Fall, 2005 Math 115 Exam) Suppose

$$f(x) = \begin{cases} e^{\sin(x)} & \text{if } x < \frac{\pi}{2} \\ kx & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

   (a) If f is continuous, what is the value of k?

   (b) Compute the average rate of change of f between $x = 1.5$ and $x = \frac{\pi}{2}$.

   (c) Compute the average rate of change of f between $x = 1.57$ and $x = \frac{\pi}{2}$.

   (d) Do you think f is differentiable at $x = \frac{\pi}{2}$?

5. Erin has noticed that her tastes have changed some since last year. Back then she spent about 15 hours a week playing softball, and 10 hours dancing. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week playing softball and 5 hours a week dancing.

   Let $S(t)$ be the number of hours Erin spent playing softball in week t, and let $D(t)$ be the number of hours she spent dancing. Assume $S(t)$ and $D(t)$ are continuous functions of time.

   (a) Are $S(t) + D(t)$, $S(t) - D(t)$, and $S(t)D(t)$ continuous?

   (b) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Erin was spending the same amount of time on softball and dance.

6. Last time we found a way to construct a fair die for any even number of sides. Prove that it's possible to construct a fair 5-sided die. Some conditions:

   - All 5 sides must be flat.
   - No handles (ala a dradle) allowed.