Douglass Houghton Workshop, Section 1, Wed 9/14/11

Worksheet Chocolate Frosting

1. As we know, Erica is an aspiring figure skater, and her idol is 2-time national champion Alissa Czisny, whose second cousin, Alex Novo, coincidently, is a former DHSP student. Once she expressed her admiration, her family began giving her items with Alissa Czisny's picture on them. She has Alissa Czisny T-shirts, an Alissa Czisny lunch box, Alissa Czisny skates, Alissa Czisny decals for her bike, and more. Right now, in the year 2011, she has 40 Alissa Czisny items. Even though she is now a little tired of Alissa Czisny, they keep coming.

   For each of the following scenarios, find how many Alissa Czisny items Erica will have t years from now.

   (a) Each year she gets 7 new items.

   (b) In year t she gets one new item for each pair she had in year $t-1$.

   (c) She gets one new item in 2012, two new in 2013, 3 new in 2014, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\mathrm{m}^2$, the towel is $T\,\mathrm{m}^2$, and he starts with 1 liter of water on him, we have

$$\text{wetness after regular toweling} = \frac{1}{1+T}$$

$$\text{wetness after ``split'' toweling} = \frac{1}{(1+T/2)^2}.$$

   Let's see just how much this "splitting" idea will buy us.

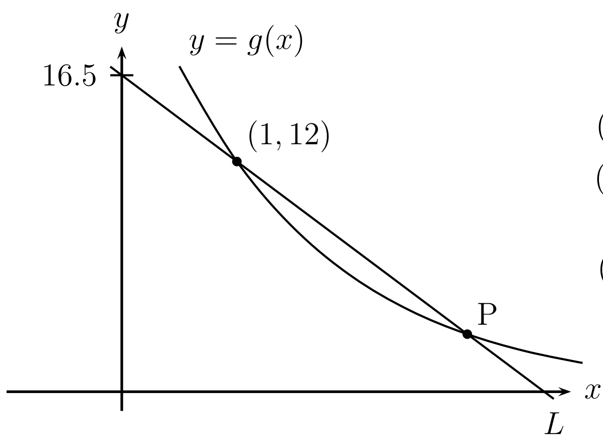
   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\mathrm{m}^2$					
$2\,\mathrm{m}^2$					
$4\,\mathrm{m}^2$					
$\frac{1}{2}\,\mathrm{m}^2$					

   (c) Consider the $1\,\mathrm{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. Consider the following graph:

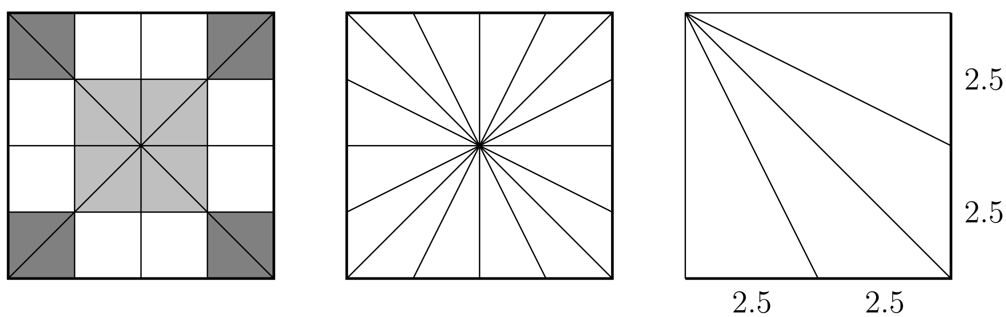


(a) Find an equation for the line L.

(b) Find the y-coordinate of the point P, given that its x-coordinate is 3.

(c) $g(x)$ is an exponential function. Find a formula for $g(x)$.

4. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a yellow cake with chocolate frosting. It's getting a bit drippy while we decide how to cut it. Last time there were two solutions for dividing the cake among 16 people:



In the first scenario, everyone gets either a white square or one dark gray triangle and one light gray triangle.

The other plan is shown in the center. Everyone gets one triangle. The picture on the right is a blow-up of the lower right corner. Not everyone was convinced that it works, because there are two different shapes of pieces.

We'd like to generalize to any number of people.

(a) Prove that both scenarios work for 16.

(b) How can they be generalized to 32 people?

(c) How about 12 people?

5. Suppose $N(p) = -.1p^2 + 13p - 62.5$ represents the profit (in dollars) that a vendor earns each day selling lemonade on the diag for p cents per cup.

(a) Graph $N(p)$ as accurately as you can, without using a calculator.

(b) What price should the vendor charge to maximize her profit? How much will the profit be?

(c) Solve $N(p) = 0$. What does your answer mean?