Worksheet Beatles

1. On Wednesday we were undecided as to whether it is always possible to dig a square hole in any triangle. Consider this particular triangle, which has been placed on a grid of unit squares:

   Can we dig a square hole in such a way that all four corners of the square are on the edges of the triangle? Explain why or why not.

2. Let $f(x) = \ln(x)$. Sketch graphs of the following, without using a calculator:

$$f(x), \quad |f(x)|, \quad f(x+2), \quad f(3x-1), \quad e^{f(x)}$$

   Which of these functions has an inverse, without restricting its domain?

3. Last time we found that when Michael Phelps dries himself off with a towel the same size as he is, then he ends up half as wet as he was before. And if the towel is twice as big as he is, he's a third as wet. But then we realized that if he just divides the big towel in half, he can get even drier.

   (a) Assume again that Michael's surface area is 1 m^2, and he starts with 1 liter of water on him. Fill in the table below with his wetness after drying in the normal way, and after drying by splitting the towel:

Towel size	Wetness after "normal" towelling	Wetness after "split" toweling
1 m^2	1/2 ℓ	
2 m^2	1/3 ℓ	1/4 ℓ
4 m^2		
8 m^2		

   (b) What if the towel has surface area T?

   (c) Can you get him even drier with the same towel?

4. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. Last time we figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:
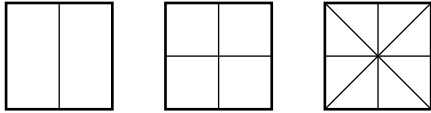
   (a) Also, a solution was proposed for $n = 16$. See if you can convince yourself (and your group) that it works.

   (b) What other numbers can you do? Explain exactly how to cut the cake and why it works.

5. Last time we showed that if the temperature in Celcius is c, then the temperature in Fahrenheit is $\frac{9}{5}c + 32$. Imagine yourself in a cabin in the backwoods of northern Scotland. You are very cold from a long day of golfing in the rain, and very tired from eating too much haggis. You need to know the temperature in Fahrenheit, so you can tell whether or not to bring the sheep in for the night. Unfortunately, this is Europe, so the only thermometer available is in Celcius, and you are too cold to multiply or divide, so you can't use the formula directly. In the cabin with you are

- a roll of duct tape
- a $33\frac{1}{3}$ RPM record player
- a Beatles record from 1967, and
- a stopwatch

among other things. How can you compute the temperature in Fahrenheit without multiplying or dividing?

6. The security detail at the Pentagon in Arlington, VA, has chained five Rottweilers to each of the five corners of the building. Each chain is 100 ft. long. Find the length of the path a single Rottweiler can walk, from one wall to the other, if she is at the end of her chain.