C-gears in rotorcraft transmissions: a novel design paradigm

Hani A. Arafa
Professor
Department of Mechanical Design and Production Engineering, Cairo University
Cairo 12316, Egypt
hani_arafa41@yahoo.com

Mostafa Bedewy†
Assistant Lecturer
Department of Mechanical Design and Production Engineering, Cairo University
† Presently at the University of Michigan, Ann Arbor, MI
mbedewy@umich.edu

ABSTRACT
This paper focuses on the final reduction stage of heavy-lift rotorcraft transmissions, typically constructed as a double-helical bull gear surrounded by several slender pinions in split-power paths from each engine. These gears are herein suggested to be re-conceived as curved-tooth gears, C-gears for short. An enumeration and a classification of the C-gear types are presented, the more promising ones are singled out, and an example is chosen to explain the mechanism of self-alignment. The inherent, omni-directional yet limited self-aligning capability, hence the alleviation of the localized edge-loading, and the absence of a center recess are the major advantages of C-gears, which are expected to not only improve the reliability, but also yield an appreciable weight saving of the transmission. This is due to the closer-to-unity load-distribution factor that can be used in the design calculations, as well as the smaller face width, hence all the associated dimensions.

INTRODUCTION
The main transmissions of heavy-lift rotorcraft as well as today’s horizontal-axis wind turbine generators have in common that they are multiple-power-path, 2.5+ MW gearboxes that run in rolling-element bearings, and both machines should run reliably while possessing the minimum possible weight, for obvious reasons in both cases. These machines have a high transmission ratio of 50-100, they operate in a converse sense, but at rotational speeds separated by an order of magnitude; the least rotor speed of helicopters (excluding the world’s largest one) is 180 rpm, and that of wind turbines is 18 rpm. Thus, a wind turbine torque of some million N.m is contrasted to several hundred-thousand N.m in a helicopter mast. Such high torques at the low-speed end pose challenging design problems, which have to be carefully dealt with, even already at the conceptual design stage. Then acceptable performance of a newly designed gearbox could be very deceptive regarding its longer-term operational behavior; the devil is in the detail, and premature failure is often imminent. This juxtaposition is brought forward because neither of the two machine categories has escaped gearbox failures, with their catastrophic consequences. This paper is thus, in a sense, on the notion of reliability by design; it presents the mechanical-design point of view on safeguarding against unexpected failure.

THE LOAD-SHARING AND LOAD-DISTRIBUTION PROBLEMS
In the past, gear failure has usually been blamed on inferior material properties, metallurgical flaws, manufacturing defects, and the inadequacy of gear design codes and factors included therein. However, it is now being observed – at an ever increasing rate – that the root cause of gearbox failure is alleged on the adverse loading conditions of the teeth. The problem areas can be classified at two levels: the unequal load sharing in multiple-power-path transmissions (including the planetary ones) and, more importantly, the tooth edge loading; the high localized stress that accelerates pitting at the tooth edge and fractures end portions of the teeth. Edge-loaded gears are noisy too.

Load-sharing equity
Many helicopter transmissions comprise a planetary gear set as a final reduction stage, typically with the planet carrier as the output member. Some of these designs have the planets supported on cantilevered pins, with an implicit aim to achieve equal load sharing by virtue of the circumferential compliance of the pins, where spherical roller bearings should take the deflection and keep the planets aligned. But a “conventional” bearing with its female spherical outer race inside the planet will cyclically rub the loaded rollers in lateral direction at the frequency of the planet/pin rotational speed, rather than act as a quasi-static self-aligning element. The deceptive fulfillment of functional requirements by this bearing arrangement and abusing the bearings as such have been discussed in the context of mechanical design “pitfalls”, referring in particular to helicopter transmissions [1]. A Solution to this problem would have been to use bearings having spherical concave rollers (the so-called design inversion) instead, with the male spherical surface on the inner race. But these bearings are yet to be seen in helicopter planetary sets.

Another solution to the combined problem of load sharing equity and planet alignment was recently reported for wind-turbine gearboxes [2, 3]. Each planet is supported on a pair of (non-self-aligning) tapered roller bearings, which are mounted on a system of two flexible cantilevers in
reverted series; a sleeve fixed at its end to the end of a pin, which is bolted at its other end to a one-sided planet carrier. The system must be such precisely calculated as to give two equal and opposite cantilever-end deflection angles, also taking the carrier-plate deflection into consideration.

In non-planetary transmissions equal load sharing can be achieved by fine phase adjustments in addition to using elastic or, better, elastomeric interface parts at the torque-splitting locations. Research is currently directed at developing better torque-splitting devices.

The bull gear trend

In conventional planetary gearboxes of multi-engine helicopters the power outputs of all engines are combined at an early stage of the drive train. Therefore, all the downstream stages are sized to transmit the gross power. It is known that, in order to keep the components of a multi-stage speed reducer as compact and lightly loaded as possible, the shaft speeds should be kept reasonably high in the initial stages then reduced to the required value in the final stage. A typical transmission [4] “adopts a double-helical gear at the output stage” which “allows a speed ratio about twice that of a simple planetary unit”; an expression of the importance of the principle stated above.

The requirement of ever increasing power density led to a clearly observed trend of using as many small final-drive pinions as possible, to be positioned around a large combining or bull gear. Up to four final-drive pinions in split-power paths from each engine are used, which provide excellent anchorage to the fuselage and balance out the radial load on the bull-gear bearings as well, Figure 1. This was obviously the best that could be done for the world’s heaviest helicopter with a rotor torque of over a million N.m from eight pinions, each under teen-tons of load [5]. This configuration also keeps the two engines substantially parallel, with no need for direction changes in the engine-to-gearbox drivelines and their individual lubrication and cooling issues. Using twelve pinions around a double-helical bull gear in three-engine helicopters was recently suggested [6, 7]. Worth mentioning is that this trend is being paralleled in the wind-turbine technology by branching the power in one high step-up ratio from a bull gear to multiple generators that run at medium speed [8].

But there still remain the problems inherent in double-helical gears and pinions, particularly those of high ratio:

1) The axial pinion shuttling upon acceleration/deceleration when the backlashes in the two gear/pinion halves are unequal [1]. This is not a rarity in such gear sets; the reason being that each of the four sets of teeth is finish-ground in a separate process.

2) The direct weight penalty due to the presence of the center recess or apex gap, which is necessary for the hobbing and grinding processes, in addition to the indirect component, which is due to stiffness-related reasons.

3) The non-uniform load distribution across the gear face; the edge-loading problem.

Figure 1. Schematics of rotorcraft transmissions with four final-drive pinions per engine to a double-helical bull gear; (a) according to Ref. 5, (b) according to Ref. 7.

The problem of non-uniform load distribution

In addition to manufacturing errors, the following causes contribute much to gear-tooth edge loading, particularly in high-ratio sets:

1) Twisting and bowing of slender pinions.

2) Gear-rim compliance (cupping) under the radial tooth-load component, particularly in thin-rim, single-webbed bull gears, which results in tooth edge loading towards the center recess.
3) Deformation of the transmission casing under load, as well as due to thermal gradients.

4) The rolling bearings, whenever they start to develop some looseness.

Crowning the tooth flanks is known to be effective only in gears of relatively small face width; candidates thus also include face gears with their cylindrical or tapered pinions [9]. Crowning could not be adopted in the bull gear/pinions of helicopter transmissions due to their relatively large face-width-to-module ratio; otherwise, load carrying capacity will be diminished. Tooth profile and lead corrections of slender pinions are generally made for the severest loading conditions; the full face will not be used in normal operation.

On the other hand, a large face-width pinion could be rendered self-aligning by being mounted on a spherical seat at its middle on a shaft, with some kinematically flexible means of torque transmittal. This arrangement could only be used with thrust- and moment-balanced pinions, viz. the spur and the double-helical. However, the problem would then convert to one of fretting, and the usually slender, high-reduction pinions will anyway prohibit such a design.

Therefore, it is deemed timely to suggest – in the present paper – a rational solution to all three problems pertaining to double-helical gears and pinions, particularly those in helicopter transmissions.

C-GEARS: GEOMETRY AND CLASSIFICATION

A pair of the so-called C-gears is shown in mesh in Figure 2. These are cylindrical, parallel-axis gears with their teeth referred to as being curved, longitudinally curved, curvilinear, circular, concavo-convex, or circoid, among other adjectives. The collective abbreviation “C-gears” has recently been suggested [10]. These gears and processes and machines for their manufacture have been invented and re-invented dozens of times, now for almost a century. For example, a most recent addition to the repertoire came from the Northwestern Polytechnic in China in 2008 [11]. The number of patents issued on C-gears and their manufacture may now exceed 100. But holding intellectual property rights by the newer inventors is very much questionable; all the C-gear geometries have been patented far earlier than 20 years ago. However, the engineering community seems to be less aware of C-gears because they have not been manufactured in volume, although an eminent gear generator manufacturer had once revealed the possibility of making C-gears on a spiral-bevel and hypoid gear cutting machine [12]. Added to this is that research publications on C-gears have been scarce, until a decade ago.

The expected self-aligning capability of C-gears, though limited, would be much more effective in preventing tooth edge-loading than tooth-flank modification across the face could be in other parallel-axis gears. The teeth will be self-conforming (in line-contact) while the pinion is left axially floating. It is herein suggested to re-conceive the double-helical final drive stage in helicopter transmissions as C-gears, which will also maintain the essential property of gradual development of contact line length and, hence, noiselessness and the benign running-in characteristics. This suggestion extends to the first stage of wind-turbine gearboxes too [13], although weight saving is not a primary design requirement in this case.

Figure 2. A pair of C-gears in mesh; numbers of teeth 25/75, face width/module ratio \( f/m = 26.5 \).

C-gears are not of one and the same kinematic geometry, there exists a considerable variety, and Figure 3 gives an enumeration and a classification thereof. Included are only the types described by exact kinematic geometry of generation; a few omissions of technically less relevant types are being made from more comprehensive listings [10, 14]. The abstract drawings in Figure 3 depict the geometry of cutting a C-rack using a straight-edge blade that rotates about the axis of a cutter head. That rotation can be continuous or oscillatory, for cutting gears with a large number of teeth and racks. This is the simplest way of describing the flank geometry of a rack, and consequently of gears with a finite number of teeth, when cut in a roll-generating process. But this description is not necessarily a
sole representative of the most appropriate method of manufacture, i.e. the real machine/tool kinematics, in all the cases. The same results could, for example, be obtained using beveled milling cutters and flaring-cup grinding wheels, where the tool will be tangent to the rack profile along one generatrix and imparted a face-traversing, arcuate motion in addition to its rotation to cut. But the possibility of using beveled milling cutters \[15\] may not – by today’s measures – be the better choice for high-speed hard cutting by coated blades.

Also shown in Figure 3 is a sub-classification into CV-gears; those in which the pressure angle varies across the face, and CC-gears; those with a constant pressure angle. In CV-gears the pressure angle increases from its value in the mid-plane (defined by the cutter-edge inclination) towards the side planes and the profiles slightly hyperbolically deviate from an involute due to the conical-curtain intersection with other than the middle transverse plane, but remain conjugate (only at the correct center distance though). These features have been described almost a century ago \[16\]. The pressure angle of CC-gears remains constant by virtue of the intersection of transverse planes with a cylindrical generating-rack swept curtain. The 3-D geometries of CC-gear tooth flanks, as generated by the two types of racks described, are so-called “involute tubes”; of slightly elliptic and circular normal sections, respectively. Each fiber of such a tube is an involute curve that stems off the base cylinder and lies in a transverse plane. Thus, the tooth trace on the unwrapped base cylinder (plane of action) and pitch cylinder (tangent plane) is elliptical and circular, respectively, for the first type, and vice versa for the latter type.

Distinction is also made between the gears generated in single-indexing (face milled), which have a symmetrical tooth trace, and those generated in continuous indexing/cutting (face hobbed) which have an unsymmetrical, prolate-trochoidal-arc tooth trace. In principle, the former types can be finish ground, while the latter types cannot. Even if – for faster production – the
gears were first cut in a continuous process for later rectification by single-indexing grinding, the deviation of the trochoidal tooth from the circular one is considered too much to be ground off.

It should be obvious that only point contact exists between two CV-gears that are both cut with a double-conical male cutter [17]. This is due to the imperative mismatch by half a pitch between the radii of curvature of the convex and concave flanks. A renowned gear manufacturer had joined the C-gear pioneers of early last century in suggesting using a female cutter head to generate the pinion; whereby the blades straddle the teeth [18]. But differently shaped gear and pinion teeth result. The latter two CV-gear types are thus not to be included in a short list of viable candidates for high-performance gearing. Even the full range of CV-gears may not be further considered, owing to the variation of the pressure angle across the face, which makes a gear pair sensitive to slight changes in the center distance; losing one of the basic merits of involute gearing. The short list is suggested to contain the two types of CC-gears, although, it must be mentioned, no viable machine/tool kinematics yet exists for their manufacture. A one-of-a-kind machine for generating cutting and grinding the latter type of CC-gears in Figure 3 was proposed in 1960 [15], but its concept is considered too long-winded, despite that it does away with the limitation on the maximum number of teeth.

TOLERANCE OF MISALIGNMENT

The main advantage of C-gears of major relevance to rotorcraft transmissions is their inherent self-aligning capability, even if only to a limited extent. This feature has long been identified and explicitly mentioned in the literature [19-21]. Experimental verification of the “ideal self-centering action” that instantly took place “when the gears were set to rotation from a condition of tooth-end-bearing” was also made [21]. This was achieved with gears having a tooth trace inclination at the side planes ($\beta$) of about 20’ with one of the gears supported in journal bearings. But geometric details pertaining to self-alignment are still missing from the literature, and will be explained hereinafter.

### Figure 4. C-rack teeth with oblique-cylinder flanks and a face advance of one pitch; contact arcs when driven crest-forward by a pinion.

#### Choice of example

An example will be used of the contact between a CC-rack with oblique-cylinder flanks, as depicted in Figure 4, and its conjugate pinion (not shown). Only half the face width is projected in top view, where all the lines are circular arcs of the same radius $R$; the tooth trace (on the pitch plane) being an identical one. This type is chosen for its simple geometry, regardless of manufacturability issues. All the machine/tool kinematics hitherto suggested, although different in the detail, are based on maintaining a V-cutter
face in a plane perpendicular to the gear axis in order for its edges to trace out the two oblique cylinders simultaneously. Ideas of using cutter shanks that are made to rotate in their cutter head in the opposite sense to satisfy that functional requirement have been suggested [22, 23]. The concept of using a cutter in reciprocating, arcuate, parallel motion has been among the very early suggestions [24], but exactly the same was re-invented a three-quarter-century later [25]. Anyway, this kinematics results in dubious cutting conditions; the rake angles vary between positive $\beta$, zero, and negative $\beta$ across the face width. Thus, still much is to be done to devise viable generating cutting and grinding mechanisms for CC-gears.

**Contact arcs**

The graphical construction in Figure 4 is rendered less ambiguous by choosing a face advance of one pitch. The pinion is meant to rotate clockwise, driving the rack to the left. The plane of action is inclined at the pressure angle $\phi$, being normal to the rack teeth profiles in all transverse planes. The plane of action, projected in front view as a line, is shown in two distinct relative positions.

Position 1 is chosen for the plane of action to intersect the concave flank of a receding tooth (#1) in the mid-plane at the depth of the form plane, giving the lowermost and last contact point. The locus of intersections at all the transverse planes gives the full-face, planar contact arc 1.1. This arc is the oblique projection, onto the rack, of the already elliptic contact arc in the plane of action; it is also an ellipse that can be very closely approximated by a circular arc of radius $R/\cos^2\phi$ within the limited face width. Intersection with the next flank is along arc 2.1, which terminates where the concave tooth tip intersects the plane of action, spanning $0.82472$ the face width in circumferential projection (calculated).

In position 2 the plane of action is shifted by a half-pitch, where it intersects three consecutive flanks. This gives full-face contact on tooth #2 (arc 2.2), in addition to contact on the receding tooth (arcs 1.2), which diminished to $0.22924$ the face width, and contact on the approaching tooth #3 (arc 3.2), which spread to $0.41543$ the face width around the mid-plane. The total circumferentially projected length of contact arcs thus changes between $1.64467$ and $1.82472$ times the face width. An indication is thus given of the contact ratio as well as the smoothness of development/transformation of the contact lines, which is effected when the face width-to-module ratio is reasonably large, as already seen in Figure 2.

**Center of misalignment**

A pair of C-gears would be expected to have relative mobility, in a skewing sense, about the center of “a” contact arc. The fact that two or three contact arcs exist simultaneously, each with its own center of curvature, poses some concern about such mobility. The limitedness of the range of self-alignment would already be suspected from a first glance at the drawing of C-gears in mesh in Figure 2. However, the range deemed sufficient to prevent tooth-edge loading is quite small, and it will be a trade-off between long contact arcs and unlimited mobility that comes into play. Therefore, in order to take full advantage of the self-aligning capability of C-gears throughout full engagement cycles, we have

1) The radius of curvature of the flanks should not be too tight relative to the face width; the proportions given in the legend in Figure 4 represent a well-balanced set, where it is also seen that the radius of curvature of the contact arcs is already larger than $R$.

2) A very small mismatch between the radii of curvature of the convex and concave flanks is advisable.

On the other hand, no “lead” correction for the windup of slender C-pinions would be necessary; the curved contact arcs will retain much the same curvature, with a slight axial shift.

**In-plane self-alignment**

In-plane misalignment of the axes of a pair of C-gears can also be tolerated, exploiting the pinion axial mobility. This can be explained on a C-pinion meshing with a gear of a large number of teeth, where the contact will resemble a sphere-in-cylinder pair; with four degrees of freedom (DOF). These are the pinion rotation (proper), the sliding along the flanks in the sense of center-distance change, and the two orthogonal components of self-alignment. Such dexterity puts aside the possible ambiguity of exactly in which plane misalignment could be tolerated, viz. the tangential plane, the plane of action, or that of the axes. It should be added that, when it comes to quasi-exact-constraint design, the 4-DOF C-gears represent an important advantage over the 3-DOF spur and helical gears and the only 2-DOF double-helical gears. It was as early as in the 1930’s that such omni-directional tolerance of misalignment of C-gears had been identified [19].

**WEIGHT SAVING**

Increasing the power density of rotorcraft transmissions has been and still is subject of extensive efforts. These include now-established measures as well as more recent attempts and suggestions, which are summarized in the following.

1) Engineering new materials for gears (and bearings) is considered to have matured, so that improvements today are incremental. The ceiling of the AGMA allowable contact stress number remains at 275 ksi (1896 MPa) for the best available carburizing and case-hardening steel grades, and this value has recently been experimentally re-affirmed also for tapered spur involute pinions in mesh with face gears [9]. Breakthroughs may or may not be imminent, and transmission designers would not feel comfortable waiting for such.
2) The adoption of split-power-path configurations with as many final-drive pinions as possible to drive a large combining gear has proven to contribute to weight saving, more significantly in the larger helicopters.

3) The relevance of the weight-saving problem is further evidenced by recent suggestions to eliminate the center recess in double-helical gear sets; coming up with pseudo-herringbone assemblies [26, 27]. But the teeth remain discontinuous.

Any further weight saving – no matter how small – will be much appreciated. The present paper suggests that “mechanical design” could again be the resort for a sensible contribution in that respect: by using C-gears, at least in the final-reduction, multiple-pinion stage. The multiple, partly interlinked features of C-gears that lead to higher power density and improved reliability of transmissions are qualitatively summarized in Figure 5. The direct weight saving through eliminating the center recess was estimated at 3% [14], while the possible adoption of a closer-to-unity load-distribution factor is estimated to save at least a further 7% of the weight. The contribution of a C-pinion, being shorter and its teeth being continuous, towards reducing its bowing under load should not be underestimated. In addition, the alleviation of the gear tooth edge-loading problem should raise the reliability/robustness of the transmission.

**CONCLUSION**

It is anticipated that eliminating a notable amount of the weight penalty and mitigating or even doing away with the gear-tooth edge-loading problem in rotorcraft transmissions could be achieved by using those 4-DOF curved-tooth gears, which action represents a novel design paradigm for these aircrafts with projected enhancements in their performance, reliability and safety. Particularly suggested herein are CC-gears; those in which the pressure angle remains constant across the face width. These gears have long ago been described, but they may only now be valued in the context of those two targets, viz. weight saving and self-alignment. Despite the apparent simplicity of the generating-rack tooth geometry, serious machine/tool kinematics is needed for generating-cutting and finishing CC-gears. This could partly be the reason why the application of these gears is still to be awaited. The present paper could stimulate efforts toward this end.

**REFERENCES**


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