Publication List

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- The strongest publications are probably 2, 7, 24, 34, and 37.
- The next tier of papers is probably 5, 20, 31, 32, 43, and:
- 11, 13, 19, 33, 36, 37 are all part of a program of studying Scott sentences of countable structures. I wrote a survey paper 3 on this area and have given a number of talks on it including a plenary talk at the 2019 logic colloquium.

Publications

1. *Computable Stone spaces* (with Nikolay Bazhenov and Alexander Melnikov)
   Submitted for publication.

   This paper is about the computable metrizability of Polish spaces up to homeomorphism. There are three main results: (1) There is a computable topological Polish space not homeomorphic to any computably metrized space. (2) We introduce a new notion of effective categoricity for effectively compact spaces and prove that effectively categorical Stone spaces are exactly the duals of computably categorical Boolean algebras. (3) We prove that, for a Stone space X, the Banach space $C(X; R)$ has a computable presentation if, and only if, X is homeomorphic to a computably metrized space.

2. *An arithmetic analysis of closed surfaces* (with Alexander Melnikov).
   Submitted for publication.

   Given two different presentations (separable metric spaces) of some compact surface, we show that there is an arithmetical isomorphism between the two. Essentially this gives a characterization of $[0, 1] \times [0, 1]$ in terms of gridlines given by Jordan curves, and shows that this can be made arithmetic by approximating the Jordan curves by sequences of points that are made more and more fine. As a consequence, all of the ways one might try to present such spaces (e.g., as simplicial complexes, embedded surfaces, etc.) are arithmetically equivalent.

3. *An introduction to the Scott complexity of countable structures and a survey of recent results.*
   Bulletin of Symbolic Logic, 28 (2022), no. 1, 71–103.

This is a survey of results on Scott sentences of countable structures and their complexity.

4. *An analysis of random elections with large numbers of voters.*
   Mathematical Social Sciences, to appear.

   This is a paper on social choice theory. In an election where each voter cast a ballot that ranks all of the candidates, we can look at the head-to-head results between each pair of candidates and form directed graph. A central issue in developing voting methods is that there can be cycles in this graph, where candidate A defeats candidate B, B defeats C, and C defeats A. Different voting methods try to solve these issues in different ways. To analyse these voting methods, we want to understand how often certain situations occur. This paper looks at random elections with the number of voters going to infinity, and show that less cyclic elections are more likely.

5. *A minimal degree low for speed* (with Rod Downey and Laurent Bienvenu).
   Journal of Symbolic Logic, to appear.

   A set is low-for-speed if using it as an oracle does not speed up (by more than a polynomial amount) the computation of any set which was already computable. Existing constructions of minimal degrees and sets low-for-speed are difficult to combine. We give a very complicated construction of a minimal degree which is low for speed; it has the interesting property that it uses e-splitting trees with the property that any two paths e-split, but there are nodes whose children do not e-split. The e-splitting is delayed.

6. *Non-density in punctual computability* (with Noam Greenberg, Alexander Melnikov, and Dan Turetsky).
   Annals of Pure and Applied Logic, 172 (2021), no. 9, 102985, 17 pp.

   This is a paper on primitive recursive computable structure theory. In traditional computable structure theory, being computably isomorphic puts an equivalence relation on the computable copies of a structure. With primitive recursive isomorphisms, the inverse of a primitive recursive map is not neccesarily primitive recursive, and so instead of an equivalence relation, we get a partial pre-ordering of "there is a primitive recurisve isomorphism from $\mathcal{A}$ to $\mathcal{B}$". In this paper we show that it is possible to have $\mathcal{A}$ below $\mathcal{B}$ in the ordering, but to have no other structures in between. We have submitted this to APAL.

7. *Computing sets from all infinite subsets* (with Noam Greenberg, Ludovic Patey, and Dan Turetsky).
   Transactions of the AMS, 374 (2021), 8131–8160.

   We say that a set A is introreducible to a set B if every infinite subset of B computes A. This is something that was studied by Jockusch in the 60's. We answer some questions that have been open since then: the collection of self-introreducible sets is Π^1_1-complete, and every introenumerable set has an introcomputable subset.

8. *Enumerations of families up to finite differences* (with Noam Greenberg, Joe Miller, and Dan Turetsky).
   Submitted for publication.

   We make some progress towards answering the following question: Is there a family of sets closed under finite differences which has Slaman-Wehner degree spectrum? We show that any such family would have to include only infinite sets, and that the construction of the set from a non-computable degree cannot be uniform. However we show that there are such families with non-low degree spectrum, and families of c.e. sets computable from a Δ^0_2 degree and from any list of non-computable c.e. degrees.

9. *Relativizing computable categoricity* (with Rod Downey and Alexander Melnikov).
   Proceedings of the AMS, 149 (2021), no. 9, 3999–4013.

   We show that there can be some weird behaviour with computable categoricity after relativizing: there are structures which switch between being computably categorical

10. *The property "arithmetic-is-recusive" on a cone* (with Uri Andrews and Noah Schweber).
    Submitted for publication.
    Journal of Mathematical Logic, 21 (2021), no. 3, 2150021.

    A first-order theory T has the property "arithmetic-is-recursive" if whenever there is an X-arithmetic model of T, that model has an X-computable copy. We show that a structure satisfies arithmetic-is-recursive on a cone if and only if it has countably many ω-bftypes. We also give an example of a theory which non-trivially has "arithmetic-is-recursive". We have submitted this to JML.

11. *Scott complexity of countable structures* (with Rachael Alvir, Noam Greenberg, and Dan Turetsky).
    Journal of Symbolic Logic, 86 (2021), no. 4, 1706–1720.

    We give a complete classification of the possible Scott complexities (simplest Scott sentence) of structures. We also show that there is a structure of Scott rank $\omega_1^{CK}+1$ which has Scott rank ω_1^{CK} after naming a constant; this was the biggest open question about structures of high Scott rank. We have submitted this to the JSL.

12. *There is a no simple characterization of the relatively decidable theories.*
    Submitted for publication.

    An elementary first-order theory is said to be relatively decidable if every model of the theory computes its own elementary diagram. This somehow seems related to model completeness (a theory is model complete if every formulas is equivalent to an existential formula). Chubb, Miller, and Solomon asked for a characterization of such theories, but we show that this property is Π^1_1-complete. I have submitted this to the JSL.

13. *Describing finitely presented algebraic structures.*
    Submitted for publication.

I prove some results about the Scott sentences of finitely presented algebraic structures, and particularly of finitely generated rings and finitely presented groups. I show that every finitely generated ring has a d-Σ_2 Scott sentence, and make progress towards the question of whether every finitely presented group has a d-Σ_2 Scott sentence. I have submitted this to Computability.

14. *Some questions of uniformity in algorithmic randomness* (with Laurent Bienvenu and Barbara Csima).
Journal of Symbolic Logic, 86 (2021), no. 4, 1612–1631.

    A left-c.e. real is 1-random if and only if it is the halting probability of a universal prefix-free machine. We know that there is a uniform way to go from a left-c.e. real to a machine with that halting probability, and Barmpalias and Lewis-Pye asked whether this could be done for 1-randoms. We show that there is no uniform way to go from a left-c.e. 1-random to a universal machine with that halting probability. We also answer a few other similar questions of uniformity. Submitted to JSL.

15. *Which classes of structures are both pseudo-elementary and $\mathcal{L}_{\omega_1\omega}$-elementary?* (with Will Boney, Barbara Csima and Nancy Day).
Bulletin of Symbolic Logic, to appear.

    We show that a class is both pseudo-elementary and $\mathcal{L}_{\omega_1\omega}$ if and only if it is $\mathcal{L}_{\omega_1\omega}$ but uses only infinitary conjunctions. If it is computable and only uses infinitary conjunctions, we show that it is pseudo-elementary using a single formula. A good example is the property of a graph of being disconnected, which is both pseudo-elementary and $\mathcal{L}_{\omega_1\omega}$. We submited this to BSL.

16. *A representation theorem for possibility models.*
Submitted for publication.

    This is a paper in modal logic about transformations between Kripke models and possibility models. This draws a strong connection between the traditional Kripke models and a recent program which is growing in popularity of using possibility semantics instead. Submitted to RSL.

17. *Relationships between computability-theoretic properties of problems* (with Rod Downey, Noam Greenberg, Ludovic Patey, and Dan Turetsky).
Journal of Symbolic Logic, 87 (2022), no. 1, 47–71.

    When proving a separation between Π^1_2 principles ("problems" with instances and solutions) in reverse mathematics, one often shows that weaker problem satisfies a preservation property that the stronger problem does not. For example, it might be that for any non-computable set C, any computable instance of the problem admits a solution that does not compute the set C. Such a problems admits "cone avoidance". We show various equivalences and non-equivalences between such properties of problems in a sort of "dual zoo".

18. *Computability up to homeomorphism* (with Alexander Melnikov and Keng Meng Ng).
Journal of Symbolic Logic, 85 (2020), no. 4, 1664–1686.

19. *Finitely generated groups are universal* (with Meng-Che Ho).
Annals of Pure and Applied Logic, 172 (2021), no. 1, 102855, 21 pp.

We show that every finitely generated structure is effectively bi-interpretable with a finitely generated group. This means that finitely generated groups are as complicated as finitely generated structures can possibly be. This is related to paper 6.

20. *The tree of tuples of a structure* (with Antonio Montalbán).
Journal of Symbolic Logic, 87 (2022), no. 1, 21–46.

Given a structure, one can build a tree of tuples from that structure, with the tree relation coming from extension of tuples. This tree captures the back-and-forth information of the structure and hence, for countable structures, the isomorphism type. We show that one cannot always compute a copy of the structure back from a copy of this tree. This has consequences about coding families into structures; one can code a family of sets into a structure without coding it into the Σ_1-types of the elements of the structure.

21. *The logic of comparative cardinality* (with Yifeng Deng and Wesley Holliday).
Journal of Symbolic Logic, 85 (2020), no. 3, 972–1005.

We completely axiomatize the logic of cardinality comparisons. The language lets you build set terms using intersection, union, and complement and compare the cardinalities of sets $|s| \geq |t|$ where s and t are terms. The difficulty is in reasoning about both finite sets and infinite sets at the same time, because they behave very differently.

22. *Graphs are not universal for online computability* (with Rod Downey, Iskander Kalimullin, Alexander Melnikov, and Daniel Turetsky).
Journal of Computing and System Sciences, 112 (2020), 1—12.

We show that graphs are not universal for primitive recursive structures. However structures in a language with one binary function are universal.

23. *Degrees of categoricity above limit ordinals* (with Barbara Csima, Michael Deveau, and Mohammad Hamdy).
Computability, 9 (2020), no. 2, 127–137.

We show that any degree c.e. in and above a $0^{(\alpha)}$, α a limit ordinal, is a degree of categoricity (and a few related results). This fills in one of the gaps in the known degrees of categoricity.

24. *Optimal bounds for single-source Kolmogorov extractors* (with Laurent Bienvenu and Barbara Csima).
Transactions of the AMS, 373 (2020), no. 3, 1983–2006.

We characterize the triples (α, β, k), $\alpha < \beta$, for which we can extract strings of information density β from strings of information density α using k extractors: i.e., those α, β, k for which there are computable $\Gamma_1, \ldots, \Gamma_k$ such that whenever $K(\sigma)/|\sigma| > \alpha$, $K(\Gamma(\sigma))/|\Gamma(\sigma)| > \beta$. Part of the proof passes through some new purely combinatorial results about discrepancy in hypergraphs.

25. *First-order possibility models and finitary completeness proofs.*
Review of Symbolic Logic, 12 (2019), no. 4, 637–662.

This is a paper in modal logic about using possibility models, which had previously been used for propsitional logic, for first-order logic. This extends the scope of a new program of studying possibility models.

26. *Automatic and polynomial-time algebraic structures* (with Nikolay Bazhenov, Iskander Kalimullin, Alexander Melnikov, and Keng Meng Ng).
Journal of Symbolic Logic, 84 (2019), no. 4, 1630–1669.

We answer a question of Khoussianov and Nerode from 2008 by showing that there is no characterization of the structures with an automatic presentation. The proof is an index set argument and uses methods from my paper number 13 below.

27. *Constructing decidable graphs from decidable structures* (with Nikolay Bazhenov).
Algebra and Logic, 58 (2019), no. 5, 369–382.

We show that the HKSS argument that graphs are universal also maintains decidability. This allows us to transfer a few results to the class of graphs.

28. *Characterizations of cancellable groups* (with Meng-che Ho).
Proceedings of the AMS, 147 (2019), no. 8, 3533–3545.

A torsion-free abelian group A has the cancelation property if whenever $A \oplus G \cong A \oplus H$, $G \cong H$. There is a Π^0_4 characterization from the 70's of the rank 1 groups with the cancellation property, and in the literature people write that it is hard to use the characterization to show that a group has the cancellation property. We show that it is Π^0_4 m-complete. We also show that for arbitrary torsion-free abelian groups it is Σ^1_1 m-hard, but we actually have no upper bound (other than Π^α_2 for some α). We're going to submit this to PAMS.

29. *Effective aspects of algorithmically random structures* (with Bakh Khoussainov and Daniel Turetsky).
Proceedings of the AMS, 147 (2019), no. 8, 3533–3545.

This is a paper with various results about Bakh's notion of a random structure. My main contribution was to show that these methods can be applied to groups (using methods from papers 6 and 15). The paper also contains results about things like the Turing degrees of random structures.

30. *A first-order theory of Ulm type.*
Computability, 8 (2019), no. 3-4, 347–358.

I show that there is a first-order theory that has many of the same properties as p-groups; for example, isomorphisms are Σ^1_1-complete but not Borel complete, and models low for ω_1^{CK} are determined up to isomorphism by their computable theory. This answers a question of Julia.

31. *There is no classification of the decidably presentable structures.*
Journal of Mathematical Logic, 18 (2018), no. 2.

I show that the index set of the structures with a decidable presentation is Σ^1_1 m-complete, and so there is no reasonable classification of such structures. This answers a question of Goncharov. The methods I introduced in this paper were also used, together with several coauthors, to solve a question about automatic structures (see 6).

32. *Borel functors and infinitary interpretations* (with Russell Miller and Antonio Montalbán).
Journal of Symbolic Logic, 83 (2018), no. 4, 1434–1456.

    We show that Borel functors between two structures are in correspondence with infinitary interpretations and also with (continuous) homomorphisms between the automorphism groups of the structures. See also 25.

33. *On optimal Scott sentences of finitely generated algebraic structures* (with Meng-Che Ho).
Proceedings of the AMS, 146 (2018), no. 10, 4473–4485.

    We answer a question of Julia by showing that there is a finitely generated group with no d-Σ^0_2 Scott sentence. We use some tools from combinatorial group theory like small cancellation and HNN extensions.

34. *Degree spectra of relations on a cone.*
Memoirs of the AMS, 253 (2018), no. 1208.

    Given a structure $\mathcal{A}$ with a relation R, we define the degree spectrum of R on a cone. The idea is to measure the degree spectra of natural classes, and hopefully to show that they have some kind of structures. I prove a number of results, some of which give structure, and some of which seem to avoid structure. The main ones are: (1) I show that a result of Harizanov can be used to show that there is no degree spectrum on a cone between the computable degrees and the c.e. degrees; (2) I show that there are incomparable degree spectra contained within the d.c.e. degrees, and in particular, degree spectra on a cone are not totally ordered; (3) I prove some results about degree spectra of relations on $(\omega, <)$; (4) I show that any degree spectrum on a cone that contains a non-Δ^0_2 degree must contain all of the 2-CEA degrees; this is the next analogue of Harizanov's result, but is much harder to prove.

35. *Computable valued fields.*
Archive for Mathematical Logic, 57 (2018), no. 5–6, 473–495.

    I analyze the effectiveness of some results of valued field theory, e.g. about extending valuation to a field extension. I also prove the technical results required to apply our metatheorem from 28 to show that every infinite-dimensional valued field satisfying one of two "nice" theories (e.g., ACVF) has a copy with a computable basis and a copy with no computable basis.

36. *Some new computable structures of high rank* (with Gregory Igusa and Julia Knight).
Proceedings of the AMS, 146 (2018), no. 7, 3097–3109.

We give a couple of new examples of computable structures of high Scott rank. The main result is an answer to a well-known question of Sacks and Millar: there is a computable structure of Scott rank ω_1^{CK} whose computable infinitary theory is not $\aleph_0$-categorical. The previously known examples of structures of Scott rank ω_1^{CK} were all $\aleph_0$-categorical.

37. *Scott ranks of models of a theory.*
Advances in Mathematics, 330 (2018), 109–147.

I describe this in more detail in my research statement. Given an $\mathcal{L}_{\omega_1\omega}$ sentence φ, which we think of as defining a class of structures, the Scott spectrum of φ is the set of Scott ranks of models of φ. I give a complete descriptive-set-theoretic characterization of the possible Scott spectra. I also show that for each α there are Π_2 sentences all of whose models have Scott rank α. I also solved two other open problems. These were problems which seemed resistent to the existing techniques.

38. *Left-orderable computable groups.*
Journal of Symbolic Logic, 83 (2018), no. 1, 237–255.

I answer a well-known question of Downey and Kurtz by showing that there is a computable left-orderable group no computable copy of which has a computable left-order. The proof requires some quite technical analysis of semidirect products.

39. *Inferring probability comparisons* (with Wesley Holliday and Thomas Icard).
Mathematical Social Sciences, 91 (2018), 62-–70.

This is a paper on qualitative probability. Basically the idea is that you forget the measure, and remember only the comparisons. What kinds of properties does such a relation have to have? There has been a lot of work in this area starting with Scott and there are various characterizations. We give a more constructive characterization than the existing ones.

40. *On computable field embeddings and difference closed fields* (with Alexander Melnikov and Russell Miller).
Canadian Journal of Mathematics, 69 (2017), no. 6, 1338–1363.

Given a field extension K/L, and an automorphism of L, can you extend that automorphism to K in an effective way? We prove a number of results about such problems, relating them to the existence of a splitting algorithm for K.

41. *The Gamma question for many-one degrees.*
Annals of Pure and Applied Logic, 168 (2017), no. 7, 1396–1405.

The statement of the Gamma question is a bit technical, but it essentially asks whether there is a 0-$\frac{1}{2}$-1-law for a particular operator Γ that takes sets to reals in $[0, 1]$. This operator Γ measures the coarse computability of a degree. Monin answered the Gamma question for Turing degrees by showing that there is a 0-$\frac{1}{2}$-1-law, but right around the same time I showed that for m-degrees, you can get everything between 0 and $\frac{1}{2}$ (an easy argument forbids values strictly between $\frac{1}{2}$ and 1).

42. *Preferential structures for comparative probabilistic reasoning* (with Wesley Holliday and Thomas Icard).
AAAI, (2017), 1135–1141.

This is another paper on qualitative probability. See 21. We prove a soundness and completeness theorem for a logic of qualitative probability.

43. *Computable functors and effective interpretability* (with Alexander Melnikov, Russell Miller, and Antonio Montalbán).
Journal of Symbolic Logic, 82 (2017), no. 1, 77–97.

We show that functors between two structures are in correspondence with infinitary interpretations of one in the other. See also 14.

44. *Degrees of categoricity on a cone via η-systems* (with Barbara Csima).
Journal of Symbolic Logic, 82 (2017), no. 1, 325–346.

We show that the degrees of categoricity of natural structures are exactly the iterates of the jump. This is a complicated η-system argument.

45. *A note on cancellation axioms for comparative probability* (with Wesley Holliday and Thomas Icard).
Theory and Decision, 80 (2016), no. 1, 159–166.

This is another paper on qualitative probability. See 21. We show that the generalized cancellation axiom is stronger than the standard cancellation axiom, a result has been suggested but not proved in the previous literature.

46. *Independence in computable algebra* (with Alexander Melnikov and Antonio Montalbán).
Journal of Algebra, 443 (2015), 441–468.

Many algebraic structures have some kind of notion of independence (such as in vector spaces or fields, or **Z**-independence in groups). A common fact for such structures is that every infinite-dimensional structure has a copy with a computable basis and a copy with no computable basis, and these two structures are Δ^0_2-isomorphic. We give a metatheorem that ties together the known results, and use the metatheorem to prove new results.

47. *Differential-algebraic jet spaces preserve internality to the constants* (with Zoé Chatzidakis and Rahim Moosa).
Journal of Symbolic Logic, 80 (2015), no. 3, 1022–1034.

This is a paper on the model theory of differential fields. There was a notion of preserving internality introduced by Moosa and Pillay, and we show that differential jet spaces (a generalization of tangent spaces) satisfy this property. The results are somewhat technical model theory, but the idea was to see whether preserving internality could happen in differential fields.

48. *Nonstandard methods for bounds in differential polynomial rings* (with Jack Klys and Rahim Moosa).
Journal of Algebra, 360 (2012), 71–86.

Given generators of an ideal, how can you decide whether the ideal is prime? One way to do this is to have a bound on the degrees of the polynomials that you have to check. Such bounds are known for fields, and we consider the problem over differential fields and attack it using ultrapowers. We show that a large number of existence-of-bounds problems are equivalent, get a new proof of the effective differential Nullstellensatz, and obtain some new bounds.