Momentum transfer using chirped standing-wave fields: Bragg scattering

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We consider momentum transfer using frequency-chirped standing-wave fields. Atom-beam splitter and mirror schemes based on Bragg scattering are presented. It is shown that a predetermined number of photon moments can be transferred to the atoms in a single interaction zone.

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Atom optics has experienced rapid advances in recent years. Applications of atom optics to inertial sensing [1], atom holography [2,3] and certain schemes for quantum computing [4,5] can benefit substantially from the ability to manipulate atomic motion in a controllable way. There are a number of theoretical and experimental studies devoted to this problem [6–9].

The underlying physical mechanism responsible for optical control of atomic motion is an exchange of momentum between the atoms and the fields. Momentum exchange can be used as the basis of practical devices, such as atom mirrors and atom-beam splitters that are essential elements of an atom interferometer. Optical π/2 and π pulses have been used to create and deflect coherent superpositional states involving different ground-state sublevels [10,11]. These experiments require one to control pulse power or duration to a fairly high precision. Alternative methods for producing large angle beam splitters involve the use of magneto-optical potentials, bichromatic forces, and strong standing wave fields [12]. Bragg scattering involving multiphoton transitions [13] can also be used to produce large angle splitting, but the power requirements increase and the resolution decreases with increasing order of the transitions.

To avoid the difficulties involved with pulses having specified areas, rapid adiabatic passage methods have been proposed [14,15]. Population transfer by stimulated Raman adiabatic passage (STIRAP) using delayed laser pulses was first observed by Bergmann and co-workers [16]. The use of this method to create an atom-beam splitter was proposed in Ref. [14]. It is worthwhile to mention that the adiabatic passage method is robust against changes in pulse parameters. Furthermore, momentum transfer based on the STIRAP technique [14,17–20] cannot be degraded as a result of spontaneous decay, as the excited states are never populated.

In this paper we propose schemes to create a 50-50 beam splitter and a mirror using frequency-chirped standing waves and adiabatic rapid passage. Both schemes are based on high-order Bragg scattering and utilize an off-resonant interaction of two-level atoms with frequency-chirped standing-wave fields. The off-resonant atom-field interaction allows us to one to avoid populating excited states; atoms remain in their ground state during the entire evolution of the atom-field interaction and spontaneous emission plays a negligible role. We use chirped pulses to produce efficient momentum transfer sequentially to states having momentum ±N 2ℏk, where k is the propagation vector of one of the fields and N is a positive integer. The chirp rate and pulse duration are used to control the final target state. This work is complementary to that involving the use of adiabatic rapid passage to accelerate atoms that are trapped in optical potentials [21].

We consider first an atom mirror, allowing us to understand the dynamics of momentum transfer. The problem is closely related to population transfer in multilevel systems, using Raman-chirped adiabatic passage [22,23], but the level scheme differs somewhat (see Fig. 3 of Ref. [13]), in that we have a doubly degenerate ladder of Bragg states. The origin of this spectrum is discussed below.

An atomic beam having longitudinal velocity \( \mathbf{u} \) in the \( \hat{x} \) direction crosses a field region in which two optical fields counterpropagate in the \( \hat{z} \) direction. The longitudinal motion is treated classically, but the transverse motion (parallel to the field propagation vectors) is quantized. The fields couple the atomic ground state [1] to an excited state [2] having energy \( E_{21} \). The Hamiltonian describing the atom-field interaction is

\[
\hat{H} = \frac{\hat{p}^2}{2m} + E_{21}|2\rangle\langle 2| - (\mu_{12}E_{12}(t)\cos[\Phi_1(t) + k\hat{z}]) \\
+ \mu_{12}E_{21}(t)\cos[\Phi_2(t) - k\hat{z}]|1\rangle\langle 2| + \text{H.c.},
\]

where \( \hat{p} \) is the center-of-mass momentum operator of the atom in the \( \hat{z} \) direction, \( \mu_{12} \) is a dipole moment matrix element, and \( m \) is the atomic mass. The laser fields have wave vectors \( \pm k\hat{z} \), pulse envelopes \( E_{\pm k}(t) \) as seen in the atomic rest frame moving with velocity \( u \) relative to the laboratory frame, and time-dependent phases \( \Phi_1(t) = \omega_0 t + \phi_1(t) \), \( \Phi_2(t) = \omega_0 t + \phi_2(t) + \delta_0 t \), where \( \omega_0 \) is a central frequency.

Assuming that the detuning, \( \Delta = E_{21}/\hbar - \omega_0 \), is large compared with \( |d[\Phi_2(t) - \Phi_1(t)]/dt| \), Rabi frequencies \( |\Omega_{\pm k}(t)| = |\mu_{12}E_{\pm k}(t)/\hbar| \), and the transverse kinetic energy term divided by \( \hbar \), we make the rotating wave approximation and adiabatically eliminate the excited-state amplitude. The equation of motion for the ground-state wave function in the momentum representation takes the following form:

\[
i\dot{a}(p,t) = \frac{p^2}{2m\hbar}a(p,t) - \Omega_c(t)\{\exp[i\phi(t)]a(p - 2\hbar k,t) \\
+ \exp[-i\phi(t)]a(p + 2\hbar k,t)\},
\]

where \( \Omega_c(t) = \Omega_k(t)\Omega_{-k}(t)/(4\Delta) \) is an effective Rabi frequency, \( \phi(t) = \phi_1(t) - \phi_2(t) - \delta_0 t \). We have omitted a factor
depending on the light shift \( \Omega_d(t) = [\Omega^2_d(t) + \Omega^2_d(t)]/(4\Delta) \) in the Eq. (2), since this corresponds simply to a redefinition of the ground-state energy.

From Eq. (2) it is clear that states with momentum \( p \) couple only to the neighboring states \( p \pm 2\hbar k \). Using the initial condition \( a(p,t=0) = \delta(p) \) one can set

\[
a(p,t) = \sum_{n=-\infty}^{\infty} a_n(t) \delta(p-2n\hbar k) \exp[\imath n\phi(t)]
\]

and write Eq. (2) in matrix form for the amplitudes \( a_n(t) \) with the Hamiltonian

\[
H(t) = \begin{pmatrix}
  \ddots & 0 & \vdots \\
  0 & E_{-1}(t)/\hbar & -\Omega_{-n}(t) & 0 \\
  -\Omega_{-n}(t) & E_{0}(t)/\hbar & -\Omega_{n}(t) & 0 \\
  0 & -\Omega_{n}(t) & E_{+1}(t)/\hbar & \ddots \\
  \end{pmatrix},
\]

where \( \Omega_{-n}(t) = \Omega_{+n}(t) = \Omega_c(t) \). The quasienergies in Eq. (3) are given by \( E_n(t) = \hbar n^2\omega_k + n\phi(t) \), where \( \omega_k = 2\hbar^2k^2/m \) is a two-photon recoil frequency, and \( n=0, \pm 1, \pm 2, \ldots \). This expression for the quasienergies is easy to understand. The Bragg levels have energy \( E_\pm = \hbar n^2\omega_k \) and the ground state \( (n=0) \) is coupled to state \( n \) by an \( n \)-two-photon process having effective frequency \( \phi(t) \). Thus, the field energy is lowered by \( n\hbar\phi(t) \) when the atom is excited to state \( n \) and this loss is reflected in the quasienergies of the atom+field. Hamiltonian (3) describes dynamics in momentum space and \( |a_n(t)|^2 \) are the probability for an atom to have momentum \( 2n\hbar k \).

In the case of an linear-chirped standing wave field, \( \phi(t) = \alpha(t-t_c) - \delta_0 \), where \( \alpha \) is the chirp rate and \( t_c \) is a constant. It is clear that in a diabatic representation there are many sequential crossings between \( E_n(t) \). They become avoided crossings owing to the interaction with the laser fields (see Fig. 1). The resonances can be viewed as sequential two-photon Bragg resonances equally spaced in time with period \( \Delta t = 2\omega_j/\alpha \). The dotted line in Fig. 1(a) represents the lowest adiabatic state. This state correlates with the zero momentum state as the pulse arrives and with the target state following the pulse. When adiabatic conditions are satisfied for all sequential crossings, atoms remain in this instantaneous eigenstate that evolves into the state having momentum \( 2n\hbar k \).

Another way to consider momentum transfer in an optical lattice is to describe motion of an atom in the periodic potential under the influence of a constant force using the Bloch formalism [24]. In this picture one can observe Bloch oscillations of the mean atomic velocity, as shown in Fig. 1(b) by dotted line. The adiabatic sequential momentum transfer mentioned above corresponds to Bloch oscillations in the lowest band. The amplitude of these oscillations is suppressed with increasing Rabi frequency (dashed line). To satisfy adiabatic conditions one must use a very small chirp rate \( \alpha \), resulting in a long period for Bloch oscillations, \( \tau_B = 2\omega_j/\alpha \), and, consequently, in a long time for momentum transfer. We show below a possibility to reduce considerably the total absolute time of momentum transfer by increasing the chirp rate. An increasing chirp rate breaks adiabaticity at the time of the first avoided crossing \( (0,1) \) [see Fig. 1(a)] and nonadiabatic couplings became important in this regime. However, one can still have efficient transfer to the target state provided \( \alpha \) is not too large. The oscillations in the solid curve of Fig. 1(b) are evidence for nonadiabatic effects; nevertheless, the transfer to the target state is nearly 100% for these parameters, as is shown below.

According to the structure of Eq. (3) the initial state \( a_0(t) \) is connected to the \( \pm 1 \) states. From the expression for the quasienergies, \( E_n(t) \), it follows that a positive chirp will sequentially bring the effective frequency into resonance with transition frequencies between states of the positive branch while states of the negative branch, will be shifted more and more from resonance [see Fig. 1(a)]. Consequently for successful momentum transfer we have to make sure that transient population of the \( -2\hbar k \) state due to the off-resonant interaction with the laser field is minimal. As we show in the example below this can be accomplished by adjusting a switching-on stage of the pulse and the chirp rate. The field must be sufficiently strong at the first crossing to insure adiabaticity. On the other hand, as the pulse is turned on, it must be sufficiently weak to avoid transitions to undesirable momentum states. It is not overly difficult to choose the pulse envelope and chirp rate to satisfy these conditions.

Figure 2 shows the momentum transfer to the \( 50\hbar k \) state for the initial condition \( a_0(t=0) = 1 \). This figure demonstrates almost 100% efficiency of adiabatic momentum trans-
fer and corresponds to an atom mirror. As a target we chose the 50\(\hbar k\) state; however, in principle, there is no limit to the number of the momentum quanta 2\(\hbar k\), which can be transferred to the atoms. As long as we approximately satisfy an adiabatic condition at the beginning of the pulse, when the field is not so strong, momentum transfer is nearly 100% efficient for the sequential Bragg resonances. The pulse duration is the parameter that determines how many transitions take place. After a target state is chosen the turn-off stage of the pulse can be adjusted as at the beginning of the pulse to avoid population of higher states.

There is enough freedom to change the time interval between sequential momentum transfers by adjusting the chirp rate. In the case of the atom mirror (see Fig. 2), the time interval between crossings, \(\Delta t = 2\omega_{1}/\alpha = 20\), in units of \(\omega_{1}^{-1}\). At the same time the Landau-Zener transition time for one of the crossings \(t_{LZ} = \Omega_{eff}(t_{i})/\alpha = 7\) where \(t_{i}\) is the crossing time. All that is required is that the transition time \(t_{LZ}\) be less than the time between crossings, \(\Delta t\). It is clear that the direction of population flow is controlled by the sign of the chirp. In our example of positive chirp, population is transferred to the positive n-mode state (the + 25th state, Fig. 2). By changing the chirp sign we are able to switch the direction of the population transfer and populate the – 25th state at a later time.

It is also possible to coherently split an atomic beam using a frequency chirped standing wave. One way to accomplish this task is to use an additional laser pulse of opposite chirp. The idea is to create simultaneously two frequency-chirped standing waves. A positively chirped standing wave provides a momentum transfer in the + \(n\hbar k\) branch of momentum states while negatively chirped wave works in parallel on the – \(n\hbar k\) branch.

In this case the modified equation for the ground-state wave function in the momentum representation takes the form

\[
\dot{a}(p,t) = \frac{p^2}{2m\hbar}a(p,t) - \Omega_{\pm}(t)a(p + 2\hbar k, t)
\]

where \(\Omega_{\pm}(t) = \Omega_{\pm}^{+}(t)\exp[i\xi_{1}(t)] + \Omega_{\pm}^{-}(t)\exp[i\tilde{\xi}_{2}(t)],\) \(\xi_{1}(t) = \phi_{1}(t) - \phi_{2}(t) - \delta_{0}\), \(\tilde{\xi}_{2}(t) = \phi_{1}(t) + \phi_{2}(t) + \delta_{0}\), \(\delta_{0}\) represents the chirp of the additional pulse, \(\Omega_{\pm}^{+}(t) = \Omega_{\pm,1}(t)/4|\Delta|,\) \(\Omega_{\pm,1}(t) = \mu_{\pm}E_{\pm}(t)/\hbar,\) \(\Omega_{\pm}^{-}(t) = \mu_{\pm}E_{\pm}(t)/\hbar\). In Eq. (4) we have omitted the common light shift.

Equation (4) can be rewritten as a linear system with the Hamiltonian as in Eq. (3). However, in this case the coupling between momentum state wave functions is different for the negative and positive branches: \(\Omega_{-}(t) = \Omega_{\pm}^{+}(t)\exp[i\tilde{\xi}_{2}(t)],\) \(\Omega_{+}(t) = \Omega_{\pm}^{+}(t)\exp[i\tilde{\xi}_{2}(t)] + \Omega_{\pm}^{-}(t),\) where \(\tilde{\xi}_{2}(t) = \phi_{23}(t) + \phi_{3}(t) + (\delta_{0} + \delta_{0})t\). More important is that the quasienergies \(E_{\pm}(t)\) and \(E_{\pm,1}(t)\) are controlled by different chirps \(\xi_{1}(t)\) and \(\tilde{\xi}_{2}(t)\): \(E_{\pm,1}(t) = E_{\pm}(t) + \hbar[\omega_{1}n^2 + n\xi_{1,2}(t)].\) As a result one can coherently control wave function dynamics in both branches of momentum states.

Figure 3 illustrates the dynamics for the case of an atom beam splitter. We choose \(\tilde{\xi}_{1}(t) = -\tilde{\xi}_{2}(t) = \alpha(t - t_{i}),\) \(E_{\pm}(t) = E_{\pm}^{+}(t) = E_{\pm}^{-}(t),\) \(\delta_{0} = \delta_{0} = 0\), and the chirp rate \(\alpha = 0.1\omega_{1}^{2}\). Our target states are + 50\(\hbar k\) and – 50\(\hbar k\). At later time we have almost perfect splitting of the population between our target states. By adjusting the efficiency of the first avoided crossing taking place between the 0 and ± 1 states in such a way that the probability of a population transfer in both directions is equal, we achieve symmetrical beam splitting. If the driving fields are derived from the same laser, a coherent superposition of the ± 50\(\hbar k\) states can be created. Note that the nonadiabatic couplings at the time of the first avoided crossings are responsible for small amount of population (≈ 0.5%) remaining in low-lying levels. We note that this scheme is very robust as well as selective and controllable. By increasing or decreasing the pulse duration by \(\Delta t = N\omega_{1}/\alpha\) (\(N\) is an integer), one can create a beam splitter of larger or smaller angle.

Let us give preliminary values of the pulse parameters that can be used in the experiments. All frequencies in our simulations have been normalized to the recoil frequency. Assuming that \(\omega_{1}/2\pi\) is about 50 kHz we find that to make a
beam splitter \((n = \pm 25)\) one has to tune the frequency difference in the range of \(50\omega_2/2\pi = 2.5\) MHz. That means a transform limited pulse of \(\approx 60\) ns duration should be used to produce the chirp \(\alpha /2\pi = 0.1\omega_2^2 \approx 1.6\) kHz/\(\mu\)s (see Figs. 2 and 3).

One method of producing appropriate chirp rates is to use acousto-optical modulators (AOM) as in studies of Landau-Zener tunneling [21]. In this case one must devise a method to transfer the temporal frequency chirp to a spatial one as an atomic beam passes through the interaction zone. One possibility for producing a spatial chirp directly is to use the Doppler shift associated with curved wave fronts [25,26].

If a Bragg beam splitter of this type is to be used as an element of an atom interferometer, the transverse momentum spread of the atomic beam must be less than some critical \(\Delta p_c\) of order \(\hbar k/(2N\omega_2 T)\), where \(T\) is the total pulse duration. This severe restriction on the momentum spread is based on the assumption that the different momentum components in the beam are uncorrelated. Each subensemble of incident atoms having initial transverse momentum \(p_c\) results in a matter grating that is roughly of the form \(\cos[2Nkz - p_cT/2m]\). When integrated over the transverse velocity distribution, the spatial grating is washed out unless the atomic beam is collimated to \(\Delta p_c \leq \hbar k/(2N\omega_2 T)\). For \(N\omega_2 T \approx 10^{-2}\), a preselection of the transverse velocities to better than \(10^{-3}\) photon recoil momenta is needed. This could be achieved by using a preparatory Bragg pulse [27,28]. Even with such extreme collimation it is still possible to have reasonable count rates. For a Na atomic beam that is first cooled transversely to a momentum spread of order \(\hbar k\), one can achieve beam densities of order \(10^{10}\) atoms/cm\(^3\). If these are longitudinally cooled to a speed of 100 cm/s and if transverse velocity selection by the preparatory pulse results in a relative loss of \(10^{-3}\), then it is still possible to have a flux on the order of \(10^{7} - 10^{8}\) atoms/s, assuming an atomic beam diameter of a few mm. For these parameters, the interaction time with a laser field having a 2 mm diameter is of the order of a millisecond or so.

One might think that, if a coherent source such as a BEC is used, the conditions on the transverse beam spread could be reduced or eliminated. This conclusion is supported by our simulations that show that the matter gratings are not degraded even if the momentum spread of the initial wave packet is as large as \(0.5\hbar k\). This is a direct demonstration of the fundamental difference between a thermal atomic beam and a coherent BEC. On the other hand, there is a significant constraint placed on the formation of gratings, using BEC’s.

To form a grating at the end of the interaction zone, it is necessary that the split components of the beam splitter still overlap. Since these components move on average with speed \(\pm 2N\hbar k/2m\) during the atom-field interaction, one requires that \((2N\hbar k T/m) < \Delta z_0\), where \(\Delta z_0\) is the initial spatial spread of the wave packet. For a minimum uncertainty packet, \(\Delta z_0 = h/2\Delta p_0\), where \(\Delta p_0\) is the initial transverse velocity spread of the BEC. As a result, one requires \(N\omega_2 T (\Delta p_0 / \hbar k) < 1\). Since \(\Delta p_0 \approx \hbar k / 100\) for typical condensates, this results in a maximum value of \(N\omega_2 T\) of order 100. More details on spatial grating analysis will be published elsewhere.

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See, for example, Atom Interferometry, edited by P.R. Berman (Academic, Cambridge, 1997).


