Aspects of hadronic physics

in the gauge/gravity correspondence

Based on:

– *Hadronic Density of States from String Theory* ,
Phys.Rev.Lett. 91(2003)111602, hep-th/0306107
with `D. Vaman`.

– *Regge Trajectories Revisited in the Gauge/Gravity Correspondence*
NPB, hep-th/0311190
with `J. Sonnenschein and D. Vaman`

talk at MIT March, 2004

Motivation:

• AdS/CFT: Beyond the Supergravity Approximation.
Sugra modes $\leftrightarrow$ Protected Operators.

• Sectors of Large Charge:

BMN: Large R-charge in $\mathcal{N}=4$ SYM $\leftrightarrow$ String shrunk to a point orbiting in S^5.

GKP: Twist-two operators $\leftrightarrow$ folded string spinning in AdS_5.

Tseytlin [integrable models]: Strings with two angular momenta in $S^5 \leftrightarrow \mathrm{Tr} Z^{J_1}\Phi^{J_2}$.

Sectors of large charges can be described by semiclassical string configurations.

Conserved classical quantities (angular momentum) $\equiv$ Quantum numbers (R-charge, spin)

Toward QCD: Look for universal quantum numbers

Properties of $\mathcal{N}=1$ SYM that are common to YM? $U(1)_R$ is broken!

Can the density of states be computed without full knowledge of the spectrum (without fully solving string theory)? [(confining) `Kutasov`]

High spin states, Regge trajectories. Hadronic states in Gauge/Gravity.

Outline

- Story of density of states in Supergravity

- Annulons thermal partition function: An exact example

- Hagedorn density of states: A semiclassical approach
(For Sugra Backgrounds dual to Confining Gauge Theories)

- Regge trajectories revisited in the Gauge/Gravity Correspondence

- The soft Pomeron trajectory: UA8 Collaboration

- Nonlinearity of Regge trajectories from string theory

Genus expansion: The intuition maker

$$Z_{string} = e^{-2\Phi_0} Z_0 + Z_1 + e^{2\Phi_0} Z_2 + \ldots$$

$$Z_{gauge} = N^2\, Z_0 + N^0\, Z_1 + \frac{1}{N^2} Z_2 + \ldots$$

- Conformal Theories: Main Contribution is N^2.
- Confining Theories: Main Contribution is N^0.

Supergravity Story Predates AdS/CFT [Klebanov]

AdS/CFT Correspondence

$AdS_5 \times S^5 \Longleftrightarrow \mathcal{N} = 4 \; SU(N)$ SYM

The Sugra limit $N \gg (g_{YM}^2 N)^{1/4} \gg 1$

$$ds^2 = h^{-1/2}(r)[-f(r)dt^2 + dx^i dx_i] + h^{1/2}[f(r)^{-1}dr^2 + r^2 d\O_5^2]$$

$$h(r) = \frac{R^4}{r^4}, \qquad f(r) = 1 - \frac{r_0^4}{r^4}$$

Temperature: $T = 1/\beta = r_0^2/\pi R^2$

$$S_{BH} = \frac{A_h}{4G} = \frac{\pi^2}{2} N^2 V_3 T^3 + \dots$$

Free $U(N)$ $\mathcal{N} = 4$ Supermultiplet

Content: Gauge Field, $6N^2$ massless scalars, $4N^2$ Weyl Fermions

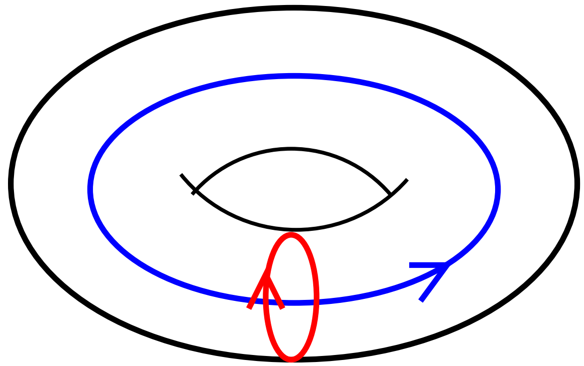
$$S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3.$$

The Famous 3/4

$$S = N^2 f(g_{YM}^2 N) V T^3$$

A proposal for a semiclassical evaluation of Z_1:

The Solitonic Object for Torus Topology World Sheet



$$X^0 = n\,\beta\,\sigma_1 + m\,\beta\,\sigma_2, \qquad \text{“Completion”}$$

Include quantum fluctuations around this soliton to compute Z_1:

$$Z \approx Z_{soliton} Z_{quantum}$$

Part I: Motivation and Justification for this proposal.

Motivating the Proposal: Compactified Boson on a Torus

- Configurations with nonzero winding number Torus $T^2 = \mathbf{C}/\Gamma : z \sim z + \omega_1 \sim z + \omega_2$

$$\Phi(z + k\omega_1 + k'\omega_2) = \Phi(z) + \beta(km + k'n), \qquad k, k' \in \mathbf{Z}$$

(m, n) Specifies a Topological configuration.

$$\Phi = \Phi^{cla}_{m,n} + \phi, \qquad \Phi^{cla}_{m,n} = \beta \left(\frac{z}{\omega_1} \frac{m\bar{\tau} - n}{\bar{\tau} - \tau} - \frac{\bar{z}}{\omega_1^*} \frac{m\tau - n}{\bar{\tau} - \tau} \right) \tag{1}$$

ϕ– periodic.

$$\begin{aligned} S[\Phi^{cla}_{m,n}] &= \frac{1}{2\pi} \int dz d\bar{z}\; \partial\Phi^{cla}_{m,n}\; \bar{\partial}\Phi^{cla}_{m,n} \\ &= \beta^2 \frac{|m\tau - n|^2}{8\pi\, \tau_2}. \end{aligned}$$

Modular Invariance $\longrightarrow$ Sum over all sectors (m, n)

$$Z = \sum_{m,n} Z_{m,n} = \frac{\beta}{\sqrt{8}\pi} \frac{1}{\tau_2^{1/2} |\eta(\tau)|^2} \sum_{m,n} \exp\left(-\frac{\beta^2}{8\pi\tau_2} |m\tau - n|^2 \right).$$

$$Z = Z_{quantum} Z_{soliton}$$

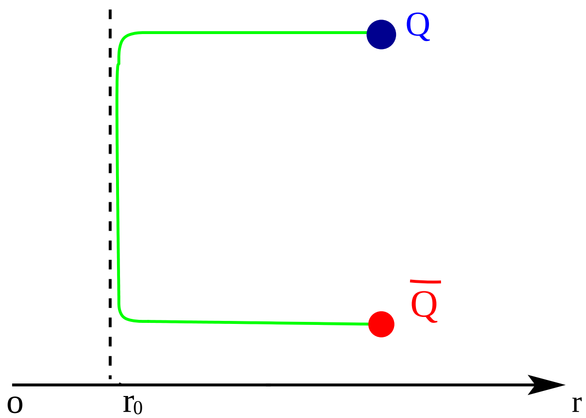
Q: Was the factorization an artifact of flat space?

Q: How to generalize for curved background?

Is there a solvable string theory of hadronic states?

Using a Penrose-Güven limit in Confining backgrounds.
Generic properties of AdS Dual of confining theories

- End of Space.
- Wilson Loop shows confining behavior.

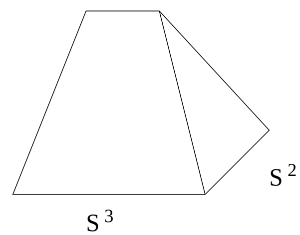


$$T_s = \frac{1}{2\pi\alpha'} g_{tt}(r_0)$$

- $g_{tt}(r_0) \neq 0$.
- $g_{tt}(r_0)$ has a minimum (`J. Sonnenschein et al.`)

The Maldacena-Núñez background

- N D5 branes wrapped on S^2.
- IR: $\mathcal{N} = 1$ SYM contaminated with KK.



$$
\begin{aligned}
ds^2_{str} &= e^{\phi_D} \left[dx_\mu dx^\mu + \alpha' g_s N (d\rho^2 + e^{2g(\rho)}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{4}\sum_a (w^a - A^a)^2) \right], \\
H^{RR} &= g_s N \left[-\frac{1}{4}(w^1 - A^1) \wedge (w^2 - A^2) \wedge (w^3 - A^3) + \frac{1}{4}\sum_a F^a \wedge (w^a - A^a) \right] \\
e^{2\phi_D} &= e^{2\phi_{D,0}} \frac{\sinh 2\rho}{2 e^{g(\rho)}}, \quad e^{\phi_{D_0}} = \sqrt{g_s N} \\
e^{2g} &= \rho \coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4}, \quad a(\rho) = \frac{2\rho}{\sinh 2\rho} \\
A &= \frac{1}{2} \left[\sigma^1 a(\rho) d\theta_1 + \sigma^2 a(\rho) \sin\theta_1 d\phi_1 + \sigma^3 \cos\theta_1 d\phi_1 \right]
\end{aligned}
$$

w^a – $SU(2)$ left-invariant one-forms

Scales associated with the $\mathcal{N} = 1$ SYM dual of the MN background.

$$
M^2_{gb} \sim M^2_{KK} \sim \frac{1}{g_s N \alpha'}, \quad T_s \propto M^2_{gb} \, (g_s N)^{\frac{3}{2}}.
$$

The Penrose-Güven limit: Set up

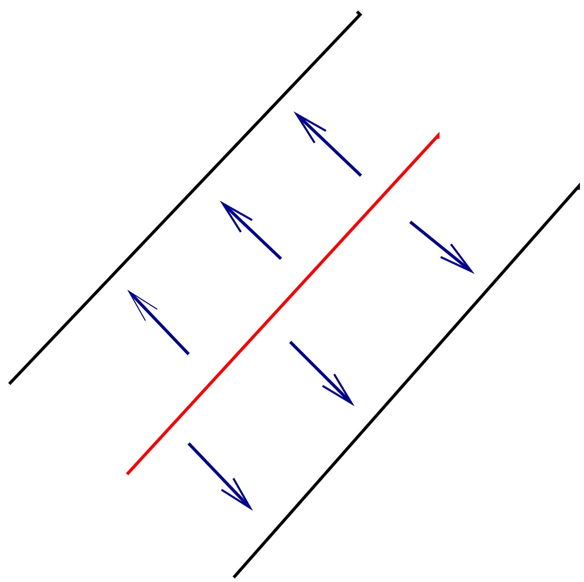
Make the following change of variables

$$dt = dx^0, \qquad x^i \to \frac{1}{L}\, x^i, \qquad \rho = \frac{m_0}{L}\, r,$$
$$\theta_2 = \frac{2\, m_0}{L}\, v, \qquad \phi_+ = \frac{1}{2}(\psi + \phi_2),$$

where $L^2 = \sqrt{g_s N}$ and $m_0 = \frac{1}{\sqrt{g_s N \alpha'}}$ is the glueball mass

$$\hat{\phi}_1 = \phi_1 + \frac{1}{3}\,\phi_+ \qquad \hat{\phi}_2 = \phi_2 - \phi_+.$$

$$x^+ = t, \qquad x^- = \frac{L^2}{2}(t - \frac{1}{m_0}\,\phi_+),$$



The Penrose-Güven Limit

$L \to \infty$; m_0 fixed

$$ds^2 = -2dx^+dx^- - m_0^2\,(\frac{1}{9}z_1^2 + \frac{1}{9}z_2^2 + v_1^2 + v_2^2)(dx^+)^2 + d\vec{x}^{\,2} + d\vec{z}^{\,2} + dv_1^2 + dv_2^2.$$

- 4 massless direction: (three x's from WV and one z).
- 2 directions (v) with mass m_0.
- 2 directions with mass $\frac{1}{3}\,m_0$.

$$H^{RR} = -2\,m_0\,dx^+ \wedge [\,dv_1 \wedge dv_2 + 1/3\,dz_1 \wedge dz_2].$$

- Fermions: 4 with mass $m_0/3$ and 4 with mass $2m_0/3$

The Hamiltonian is(Poincare time/Energy):

$$H = -p_+ = i\partial_+ = E - m_0(-\frac{1}{3}J_1 + J_2 + J_\psi) = E - m_0\,J,$$

$$P^+ = -\frac{1}{2}p_- = \frac{i}{2}\partial_- = \frac{m_0}{\Omega^2}(-\frac{1}{3}J_1 + J_2 + J_\psi) = m_0\frac{J}{\sqrt{g_sN}}.$$

The Annulon Hamiltonian [Gimon,LAPZ,Sonnenschein,Strassler]

The light-cone Hamiltonian of the theory has the following simple form:

$$
\begin{aligned}
H &= \frac{P_i^2}{2P^+} + \frac{P_4^2}{2P^+} + \frac{1}{2\alpha' P^+} \sum_{n=1}^{\infty} n(N_n^i + N_4^i) \\
&+ \frac{1}{2\alpha' p^+} \sum_{n=0}^{\infty} \left(w_n^a (N_n^1 + N_n^2) + w_n^b (N_n^3 + N_n^4) \right) \\
&+ \frac{1}{2\alpha' p^+} \sum_{n=0}^{\infty} \left(\omega_n^\alpha \mathcal{S}_n^\alpha + \omega_n^\beta \mathcal{S}_n^\beta \right) .
\end{aligned} \tag{2}
$$

where $i = 1, 2, 3, \mathbf{4}$, $a = 5, 6$, $b = 7, 8$, $\alpha = 1, 2, 3, 4$ and $\beta = 5, 6, 7, 8$; N and $\mathcal{S}$ are bosonic and fermionic occupation numbers

$$
\begin{aligned}
w_n^a &= \sqrt{n^2 + (m_0 p^+ \alpha')^2}, \qquad w_n^a = \sqrt{n^2 + \frac{1}{9}(m_0 p^+ \alpha')^2}, \\
\omega_n^\alpha &= \sqrt{n^2 + \frac{1}{9}(m_0 p^+ \alpha')^2}, \qquad \omega_n^\beta = \sqrt{n^2 + \frac{4}{9}(m_0 p^+ \alpha')^2}
\end{aligned} \tag{3}
$$

A string theory of hadrons

The Hamiltonian purely in Field Theory language

$$
H = [\frac{\mathcal{P}_i^2}{2m_0 J} + \frac{T_s}{2m_0 J} (\mathcal{N}_R + \mathcal{N}_L)] + [\frac{T_s}{2m_0 J} (H_0 + H_R + H_L)] \ .
$$

Towards the MN Annulon partition function

- building blocks: Boson off criticality [Itzykson and Saleur].

$$z_{lc}^{(0,0)}(\tau,m)=\int\mathcal{D}X\exp\Big[-\int_T d^2z\bar{X}(-\partial_z\partial_{\bar{z}}+m^2)X\Big], \tag{4}$$

Doubly periodic quantum boson $z=\xi_1+\tau\xi_2$,

$$X(\xi_1,\xi_2)=\sum_{n1,n2\in\mathbf{Z}}X_{n1,n2}\exp[2\pi i(n_1\xi_1+n_2\xi_2)]$$

$$\begin{aligned} d^2z &= d\xi_1 d\xi_2\tau_2, \\ \partial_z\partial_{\bar{z}} &= \frac{1}{4{\tau_2}^2}\left(|\tau|^2{\partial_1}^2-2\tau_1\partial_1\partial_2+{\partial_2}^2\right), \end{aligned}$$

Explicit Gaussian integrals over $X_{n1,n2}$

$$z_{lc}^{(0,0)}(\tau,\mu)=\left[\prod_{n_1,n_2\in\mathbf{Z}}\tau_2\left((\frac{2\pi}{4\tau_2})^2|n_1\tau-n_2|^2+m^2\right)\right]^{-1}.$$

Double Product $\longrightarrow$ Modular properties

$$z_{lc}(-1/\tau,m|\tau|)=z_{lc}(\tau,m)$$

Fermionic Partition function [antiperiodic in ξ_1]

$$z_{lc}^{(1/2,0)}(\tau,\mu)=\prod_{n_1,n_2\in\mathbf{Z}}\tau_2\left((\frac{2\pi}{4\tau_2})^2\left|\frac{2n_1+1}{2}\tau+n_2\right|^2+\frac{\mu^2\,\beta^2}{\tau_2^2}\right)$$

Too Formal!

A Nonholomorphic Generalization of Dedekind $\eta(\tau)$

Performing one of the infinite products

$$\begin{aligned} z_{lc}^{(0,0)}(\tau, m) &= \exp\left[2\pi\tau_2\left(m/2 + \sum_{n=1}^{\infty}\sqrt{n^2+m^2}\right)\right] \\ & \left[\prod_{n\in\mathbf{Z}}\left(1 - \exp[2\pi i(\tau_1 n + i\tau_2\sqrt{n^2+m^2})]\right)\right]^{-1}. \end{aligned}$$

Compare with

$$|\eta(\tau)|^2 = \exp\left[-\pi\tau_2/6\right]\left[\prod_{n\in\mathbf{Z}}\left(1 - \exp[2\pi i n(\tau_1 + i\tau_2)]\right)\right]^{-1} \tag{5}$$

ζ-function regularization of the Casimir Energy

$$\begin{aligned} \gamma_0(m) = \frac{m}{2} + \sum_{n=1}^{\infty}\sqrt{n^2+m^2} &= \frac{m}{2} + \left[-\frac{1}{12} + \frac{1}{2}m - \frac{1}{2}m^2\ln(4\pi e^{-\gamma})\right. \\ &+ \left.\sum_{n=2}^{\infty}(-1)^n\frac{\Gamma(n-\frac{1}{2})}{n!\Gamma(-\frac{1}{2})}\zeta(2n-1)m^{2n}\right], \end{aligned}$$

γ – Euler constant. Flat space limit ($m \to 0$)

$\gamma_0(m) \longrightarrow \sum_{n=1}^{\infty} n = \zeta(-1) = -1/12.$

The MN Annulon Partition Function

$$\begin{aligned}
Z(\beta,\mu) \; = \; & \frac{\beta}{4\pi\, l_s} \int_0^\infty \frac{d\tau_2}{{\tau_2}^2} \int_{-1/2}^{1/2} d\tau_1 \sum_{r=1}^{\infty} \left[1-(-1)^r\right] \exp(-\frac{\beta^2\, r^2}{2\pi\,\alpha'\,\tau_2}) \\
\times \; & \left[{\tau_2}^{-1/2} |\eta(\tau)|^{-2}\right]^4 \qquad 4 \quad \text{massless bosons} \\
\times \; & \left[z_{lc}^{(0,0)}(\tau, \frac{m_0\, \beta\, r}{\tau_2})\right]^2 \qquad 2 \quad m_0 \text{ bosons} \\
\times \; & \left[z_{lc}^{(0,0)}(\tau, \frac{m_0/3\, \beta\, r}{\tau_2})\right]^2 \qquad 2 \quad m_0/3 \text{ bosons} \\
\times \; & \left[z_{lc}^{(1/2,0)}(\tau, \frac{m_0/3\, \beta\, r}{\tau_2})\right]^4 \qquad 4 \quad m_0/3 \text{ fermions} \\
\times \; & \left[z_{lc}^{(1/2,0)}(\tau, \frac{2m_0/3\, \beta\, r}{\tau_2})\right]^4 \qquad 4 \quad 2m_0/3 \text{ fermions} \\
+ \; & \text{stuff associated with a nonsupersymmetric ground state}
\end{aligned} \tag{6}$$

Hagedorn Temperature:

$$-\frac{T_s\beta_H^2}{2\pi}+\frac{2}{3}\pi-4\pi\gamma_0(m_0\beta_H)-4\pi\gamma_0(\frac{m_0}{3}\beta_H)+8\pi\gamma_{1/2}(\frac{m_0}{3}\beta_H)+8\pi\gamma_{1/2}(\frac{2m_0}{3}\beta_H)=0. \qquad (7)$$

Limits: $m_0 \to 0$ [IIB Strings in Flat Space]

Large m_0 a lower dimensional theory with $\beta_H = 2\pi/(\sqrt{3}T_s^{1/2})$

Density of States for the MN Annulons

$$S=\frac{2\pi}{\sqrt{3}}\frac{E}{\tilde{T}_s^{1/2}}.$$

$\tilde{T}_s = T_s/J$. In general $S(m_0, T_s/J)$.

Where are the Temporal Windings?

- Using Light-Cone, temporal coordinate gauged away, RR field.
- How can this result be turned into evidence for the proposal.

The above Partition function was written over the strip:

$$E: \quad \tau_2 > 0, \quad -\frac{1}{2} < \tau_1 < \frac{1}{2}.$$

Generalizing a flat space result: Tiling.

$$\begin{aligned} Z(\beta,\mu) \; = \; & \frac{\beta}{4\pi\, l_s} \int_{\mathcal{F}} \frac{d\tau_2}{{\tau_2}^2} \int d\tau_1 \sum_{m,n}{}' \prod_{n_1,n_2\in \mathbf{Z}} \exp\left(-\frac{\beta^2|m\tau+n|^2}{2\pi\alpha'\tau_2}\right) \\ \times \left[{\tau_2}^{-1/2}|\eta(\tau)|^{-2}\right]^4 \quad & \left[\frac{\pi^2}{4\tau_2}|n_1\tau-n_2|^2+\frac{\mu^2\,\beta^2}{\tau_2}|m\tau+n|^2\right]^{-2} \\ & \left[\frac{\pi^2}{4\tau_2}|n_1\tau-n_2|^2+\frac{\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{-2} \\ & \left[\frac{\pi^2}{4\tau_2}\left|\frac{2n_1+1}{2}\tau+n_2\right|^2+\frac{\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{4} \\ & \left[\frac{\pi^2}{4\tau_2}\left|\frac{2n_1+1}{2}\tau+n_2\right|^2+\frac{4\mu^2\,\beta^2}{9\tau_2}|m\tau+n|^2\right]^{4} \\ + \; & \text{stuff associated with a nonsupersymmetric ground state} \end{aligned} \tag{8}$$

$m, n \in \mathbf{Z}^*$

Integration over the fundamental domain:

$$\mathcal{F}: \quad |\tau| > 1, \quad -\frac{1}{2} < \tau_1 < \frac{1}{2}.$$

Mixing $Z_{quantum} Z_{soliton}$: (m, n) dependent masses:

$$\mu \to \mu |m\tau + n|$$

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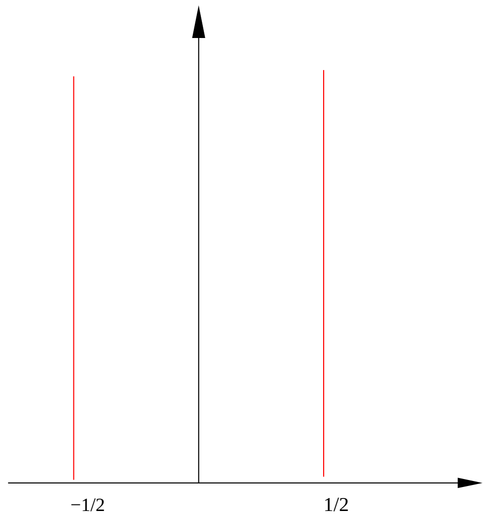
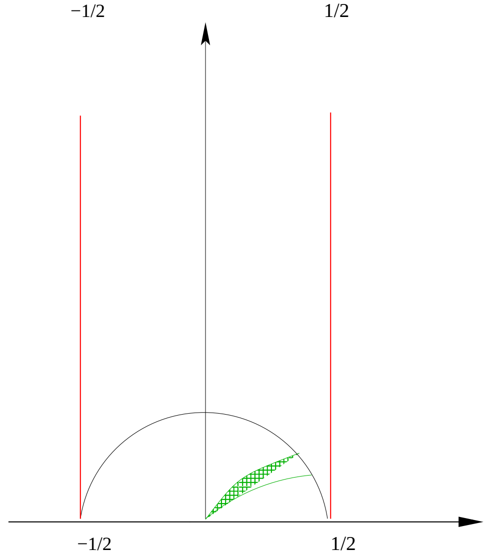




Temporal winding modes as solitons

The World Sheet has Torus Topology:

$$ds^2 = |d\sigma_1 + \tau d\sigma_2|^2 = d\sigma_1^2 + |\tau|^2 d\sigma_2^2 + 2(\mathrm{Re}\,\tau) d\sigma_1 d\sigma_2.$$

String Action [bosonic]:

$$I = \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{\gamma}\, g_{\mu\nu} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu.$$

EOM [assumption: $g_{\mu\nu}(r)$]:

$$\partial_\alpha (\sqrt{\gamma} \gamma^{\alpha\beta} g_{00} \partial_\beta X^0) = 0.$$

$$\partial_\alpha \left(\sqrt{\gamma} \gamma^{\alpha\beta} g_{rr} \partial_\beta r \right) - \frac{1}{2} \partial_r g_{00} \left[\sqrt{\gamma} \gamma^{\alpha\beta} \partial_\alpha X^0 \partial_\beta X^0 \right] = 0.$$

Looking for the winding Soliton:

$$X^0 = m\beta\sigma_1 + n\beta\sigma_2, \qquad r = r(\sigma_1, \sigma_2).$$

Complicated in General

$$\exists\, r_0 : \;\; g_{00}(r_0) \neq 0, \quad \partial_r g_{00}(r_0) = 0.$$

- For confining backgrounds $r = r_0$ is a solution [String at the AdS Wall] $T_s = g_{00}(r_0)/2\pi\alpha'$

$$S_\beta(m,n) = T_s \frac{1}{\tau_2}\Big[\beta^2(n^2 + m^2|\tau|^2 - 2(\mathrm{Re}\,\tau)\, m\, n)\Big] = T_s \frac{\beta^2|m\tau - n|^2}{\tau_2}.$$

Sketch of Fluctuations:

- r_0 is now $\tau = 0$.
- $e_3^2 + e_4^2 + e_5^2|_{\tau=0} = \frac{1}{2} d\Omega_3^2$ round S^3 with radius $1/\sqrt{2}$.
- $S^3(\theta, \phi, \psi)$ by fixing $\theta = \pi/2$ becomes $\mathbf{R}^3(y^1, y^2, y^3)$.
- $e^{2g}|_{\tau=0} \approx \tau^2 \longrightarrow \tau$-direction with $S^2(\theta_1, \phi_1)$ into $\mathbf{R}^3(\tau^1, \tau^2, \tau^3)$

$$
\begin{aligned}
S_{2b} &= S[X^0_{classical}, r = r_0] + \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{\gamma}\gamma^{\alpha\beta} \left(\partial_\alpha X^a \partial_\beta X^a g_{00} + \alpha' g_s N g_{00} [\partial_\alpha \tau^i \partial_\beta \tau_i + \frac{1}{4} \partial_\alpha y^i \partial_\beta y_i] \right. \\
&+ \left. \frac{4\beta^2}{9 Im\tau^2} g_{00} |m\tau - n|^2 \tau^i \tau_i \right)
\end{aligned} \tag{9}
$$

where $a = 1, \ldots, 4$ and $i = 1, 2, 3$.

- The mass $(2/3)\beta\sqrt{\frac{1}{\alpha' g_s N}} |m\tau - n| / Im\tau$.

$$
S_{2f} = \frac{i}{2\pi\alpha'} \int \bar{\theta}^I (\sqrt{\gamma}\gamma^{\alpha\beta}\delta^{IJ} - \epsilon^{\alpha\beta}\sigma_3^{IJ}) \partial_\alpha X^0 \Gamma_{\underline{0}} e^{\underline{0}}_0 (\delta^{JK}\partial_\beta + \frac{1}{8 \cdot 3!} e^\phi \sigma_1^{JK} \Gamma^{\mu_1\mu_2\mu_3} F_{\mu_1\mu_2\mu_3} \partial_\beta X^0 \Gamma_{\underline{0}} e^{\underline{0}}_0) \theta^K \tag{10}
$$

$F_{(3)} = -\frac{1}{4} g_s N dy_1 \wedge dy_2 \wedge dy_3$

Choose the κ gauge: $\theta^1 = \theta^2$

Hagedorn Behavior

• Partition function:

$$Z_{T^2} \Rightarrow \sum_{m,n\in\mathbf{Z}}{}' \frac{\beta}{2\pi l_s}\int_{\mathcal{F}} d^2\tau \frac{1}{Im\tau^2} e^{-\frac{\beta^2 g_{00}}{4\pi\alpha'}\frac{|m\tau-n|^2}{Im\tau}} z^b_{0,0}(\tau,0)^5 z^b_{0,0}(\tau, M^2 = \frac{4}{9}\beta^2\frac{|m\tau-n|^2}{Im\tau^2}\frac{1}{\alpha' g_s N})^3 z^f_{b_1,b_2}(\tau,0)^8 \quad (11)$$

$$Z_{T^2} \approx \int e^{-\frac{\beta^2 g_{00}}{4\pi\alpha'} Im\tau} e^{-\pi Im\tau \sum_{l\in\mathbf{Z}}(5l+3\sqrt{l^2+\frac{4}{9}\beta^2\frac{1}{\alpha' g_s N}}-8(l+\frac{1}{2}))}, \quad (12)$$

T_H:

$$\frac{1}{4\pi\alpha'}\beta_H^2 g_{00} = -2\pi\Big(5\gamma_0(0) + 3\gamma_0(2\beta_H\sqrt{\frac{1}{\alpha' g_s N}}/3) - 8\gamma_{1/2}(0)\Big). \quad (13)$$

$$d(E) \approx \exp\left(\sqrt{3\pi}\,\frac{E}{T_s^{1/2}}\right). \quad (14)$$

The Density of states depends on the gauge theory quark-antiquark string tension

Regge Trajectories Revisited in the Gauge/Gravity Correspondence

- A Regge trajectory: a line in the Chew-Frautschi plot: $J = \alpha_0 + \alpha' t$

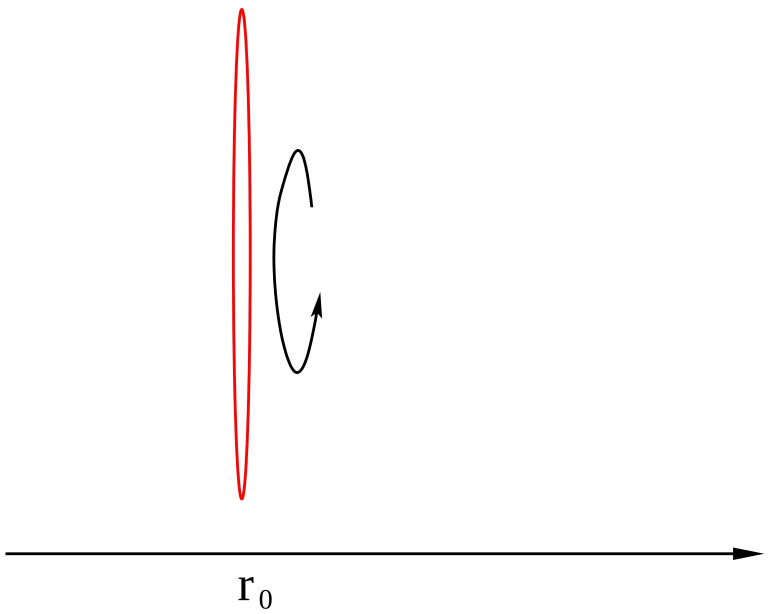
- Well described by simple strings model but *now* we have the *right* string models.

Gauge Theory State	String Theory Configuration
Glueballs	Spinning Folded Closed String
Mesons of heavy quarks	Spinning open strings ending at boundary
Baryons of heavy quarks	Strings attached to a baryonic vertex
Dibaryons	Strings attached to wrapped branes
Mesons of light quarks	Spinning open strings ending on D7

States in gauge theory and their corresponding classical configuration in the string theory

Closed spinning strings in supergravity backgrounds

Regge trajectories for Glueballs



Closed spinning strings in confining theories

$$ds^2 = h(r)^{-1/2}\Big[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\Big] + h(r)^{1/2}dr^2 + \dots \tag{15}$$

The relevant classical equations of motion are

$$\begin{aligned}
\partial_a(h^{-1/2}\eta^{ab}\partial_b t) &= 0,\\
\partial_a(h^{-1/2}\eta^{ab}\partial_b x^i) &= 0,\\
\partial_a(h^{1/2}\eta^{ab}\partial_b r) &= \frac{1}{2}\partial_r(h^{-1/2})\eta^{ab}[-\partial_a t\partial_b t + \partial_a x_i \partial_b x^i].
\end{aligned} \tag{16}$$

- Conditions for confinement in gauge/gravity: g_{00} has a nonzero minimum at some point r_0.

$$\partial_r(g_{00})|_{r=r_0} = 0, \qquad g_{00}|_{r=r_0} \neq 0. \tag{17}$$

$$t = e\,\tau, \qquad x_1 = \frac{1}{\omega}\cos e\omega\,\tau\ \sin e\omega\,\sigma \qquad x_2 = \frac{1}{\omega}\sin e\omega\,\tau\ \sin e\omega\,\sigma \tag{18}$$

$$E = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega}\frac{\pi}{2}, \qquad J = \frac{g_{00}(r_0)}{2\pi\alpha'}\frac{1}{\omega^2}\frac{\pi}{4}. \tag{19}$$

Typical Regge trajectories

$$E^2 = 4\,\pi T_s\ S, \quad \text{or} \quad \mathrm{J} = \frac{\alpha'}{2}\ \mathrm{t} \tag{20}$$

Problems

- This trajectory has zero intercept.
- It is strictly linear.

The soft Pomeron: UA8 Collaboration

- Experimental suggestion

$$\alpha(t) = 1.10 + 0.25\,t + \alpha''\,t^2,$$

$$\alpha'' = 0.079 \pm 0.012 \mathrm{GeV}^{-4}$$

- Positive Nonvanishing intercept.
- Positive curvature $\alpha'' > 0$.

The slope from sugra data

State	$(\mathrm{Mass})^2/\varepsilon^{4/3}$
0^{++}	9.78
0^{++*}	33.17
1^{--}	14.05
1^{--*}	42.90
2^{++}	18.33

- This table was obtained in 2000 [`Cácers and Hernández`], state in red 04 [`Amador and Cáceres`].
- The prediction for the Regge slope

$$J = \alpha(t) = 0.234\,t + \alpha_0$$

Semiclassical quantization

- Compute how the classical energy changes (similar to Lüscher term):

$$e\Delta E = \int d\sigma < \Psi|\mathcal{H}(\delta X)|\Psi >= \text{sum of zero} - \text{point energies}$$

- New feature of confining backgrounds? [Compared to strings in flat space]

$$\gamma^{\tau\tau} g_{tt}\partial_\tau t\partial_\tau t + \gamma^{\alpha\beta}\partial_\alpha x^i \partial_\beta x^i g_{ii} = \left(\frac{8e^{\phi_0}}{9}\ \kappa^2 \cos^2\omega\sigma\right)\ \tau^i\tau_i \tag{21}$$

$$[\partial_\tau^2 - \partial_\sigma^2 + m_0^2\cos^2(\omega\sigma)]\delta\tau_i = 0. \tag{22}$$

Mathieu differential equation

$$\lambda_{r,n} = \frac{n^2}{\omega^2} + \frac{m_0^2}{2\omega^2} + \frac{r^2}{\omega^2} + \frac{1}{2(r^2-1)}\frac{m_0^4}{16\omega^4} + \mathcal{O}(m_0^8), \tag{23}$$

- Contribution to the zero point energy

$$\Delta E = -\frac{1}{12} + m_0. \tag{24}$$

- Fermions

$$S_F \approx \frac{i}{2}T_s \int \sqrt{\gamma}\gamma^{\alpha\beta}\left(\bar{\theta}\partial_\alpha \bar{X}^\mu \Gamma_\mu \partial_\beta \theta + \frac{1}{4}\partial_\alpha \bar{X}^\mu \partial_\beta \bar{X}^\nu \bar{\theta}\gamma_\mu \hat{f}\gamma_\nu\right), \quad \hat{f}^2 = 2e^2\ell^2\cos^2\omega\sigma \tag{25}$$

Nonlinear Regge Trajectories

$$E - E_{Class} = \pi\left(\frac{3}{2}m_0 - 4\ell\right) = z_0$$

	Klebanov-Strassler	Maldacena-Núñez
m_0	$\frac{3^{1/6}a_1^{1/2}}{a_0} \quad \frac{\varepsilon^{2/3}}{g_s M\alpha'}$	$\frac{2}{3}\,\frac{1}{\sqrt{g_s N\alpha'}}$
ℓ	$\frac{3^{1/2}}{2^{7/6}\,a_0} \quad g_s^{-1}\frac{\varepsilon^{2/3}}{g_s M\alpha'}$	$\frac{2^{1/2}}{g_s N\sqrt{g_s N\alpha'}}$

$$J = \frac{1}{2}\alpha' \; E^2 - \alpha' z_0 \; E + \frac{1}{2}\alpha' z_0^2.$$

$$J \equiv \alpha(t) = \alpha_0 + \frac{1}{2}\alpha' t + \beta\sqrt{t}$$

- Positive Nonvanishing intercept.
- Positive curvature $\alpha''(t) > 0$.

Outlook:

- Exact Calculation of the Density of States in Hadronic String Theories.

- A proposal for how to compute the Hagedorn Density of States when the full string solution is not available.

- How about transitions: Confinement/Deconfinement?

- Nonzero intercept and nonlinearity of glueball trajectories.

- Regge trajectory for dynamical mesons (light masses). Description of the finer structure.

- What other hadronic properties can one get a handle on?