IPR-97-04

Study of Circular Polarisation in the Hermes Longitudinal Polarimeter

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March 27, 1997

Contents

1 Introduction

During the HERA shutdown in winter 96/97 the Longitudinal Polarimeter has been subject of a major hardware upgrade. A new set of mirrors has been installed with a special coating to preserve the polarisation of the light better. Also new lenses and a new entrance window to the laser beam pipe were put in place. During January 1997 the Longitudinal Polarimeter group has performed measurements with linear and circular polarised light to check the new components and to perform a detailed measurement of the transport of the laser light through the whole system. This note describes various experimental setups used during this phase (section 2), summarises the measurements (section 3), gives the results (section 4) and draws first conclusions (section 5). The descriptions in this note focus on the important parts for the circular polarisation. A more complete description of the Longitudinal Polarimeter can be found elsewhere [1].

2 Experimental Setup

The measurements where done at the Longitudinal Polarimeter system with the default setup and with some small modifications.

The default setup consists of the following equipment (in the order of the beam direction), see Fig. 1:

- Nd:YAG laser
- 2 mirrors (not shown in Fig. 1)
- Pockels-cell (PC)
- optional half-wave plate (not shown in Fig. 1)
- optional analyser box 1 (AB 1)
- beam expander
- laser pipe entrance window (Window 1)
- mirrors M1 to M3
- lense doublet
- mirror M4
- optional analyser box 1 before M5/6 (not shown in Fig. 1)
- mirrors M7 and M8
- HERA vacuum entrance window (Window 2)
- HERA vacuum exit window (Window 3)
- mirrors 7/8
- analyser box 2 (AB 2)

The Pockels cell produces linear, elliptical or circular polarised light, depending on the applied voltages. The optimum for circular polarisation was determined by scanning the high voltage range. With no voltage connected the Pockels cell does not affect the linear polarised light generated by the laser.

The optional half-wave plate behind the Pockels cell is used in connection with linear polarised light (i.e. Pockels cell high voltage switched off). Rotation of the half-wave plate allows to measure the transmission of linear light in different orientations through the system. In case the phase is changed through the laser beam transport system one will measure a certain fraction of circular component and the orientation might have changed.

Analyser box number 1 serves as the reference system and checks the linear and circular light after the Pockels cell and halve-wave plate. Analyser box number 2 measures the light polarisation at the end of the optical system and allows conclusions on the polarisation at the interaction point (IP) (inside the HERA vacuum). For the tests in the open system we have mounted temporary analyser box 1 before the mirrors M5/6 to have an additional data point before the interaction point.

During the measurements, window 1 was exchanged. Additionally the laser beam pipe was either vented to air or evacuated, the HERA beam pipe was either under vacuum or was vented with N_2 to atmospheric pressure. To check the effect of stress on the HERA entrance and exit windows (windows 2 and 3), the screws were untightened, while the pipe was filled with N_2. The main set of different experimental conditions is tabulated in Tab. 1.

id	entrance window 1	laser pipe	HERA window 2	HERA beam pipe	HERA window 3
A)	old	vacuum	fixed	vacuum	fixed
B)	old	vacuum	fixed	N_2	fixed
C)	old	air	fixed	N_2	fixed
D)	old	air	loose	N_2	loose
E)	removed	air	loose	N_2	loose
F)	removed	air	fixed	N_2	loose
G)	removed	air	fixed	N_2	semi fixed
H)	removed	air	fixed	N_2	fixed
I)	new	vacuum	fixed	vacuum	fixed
J)	new	vacuum	fixed	vacuum	fixed
K)	new	air	fixed	vacuum	fixed

Table 1: Conditions for linear scans and high voltage scans.

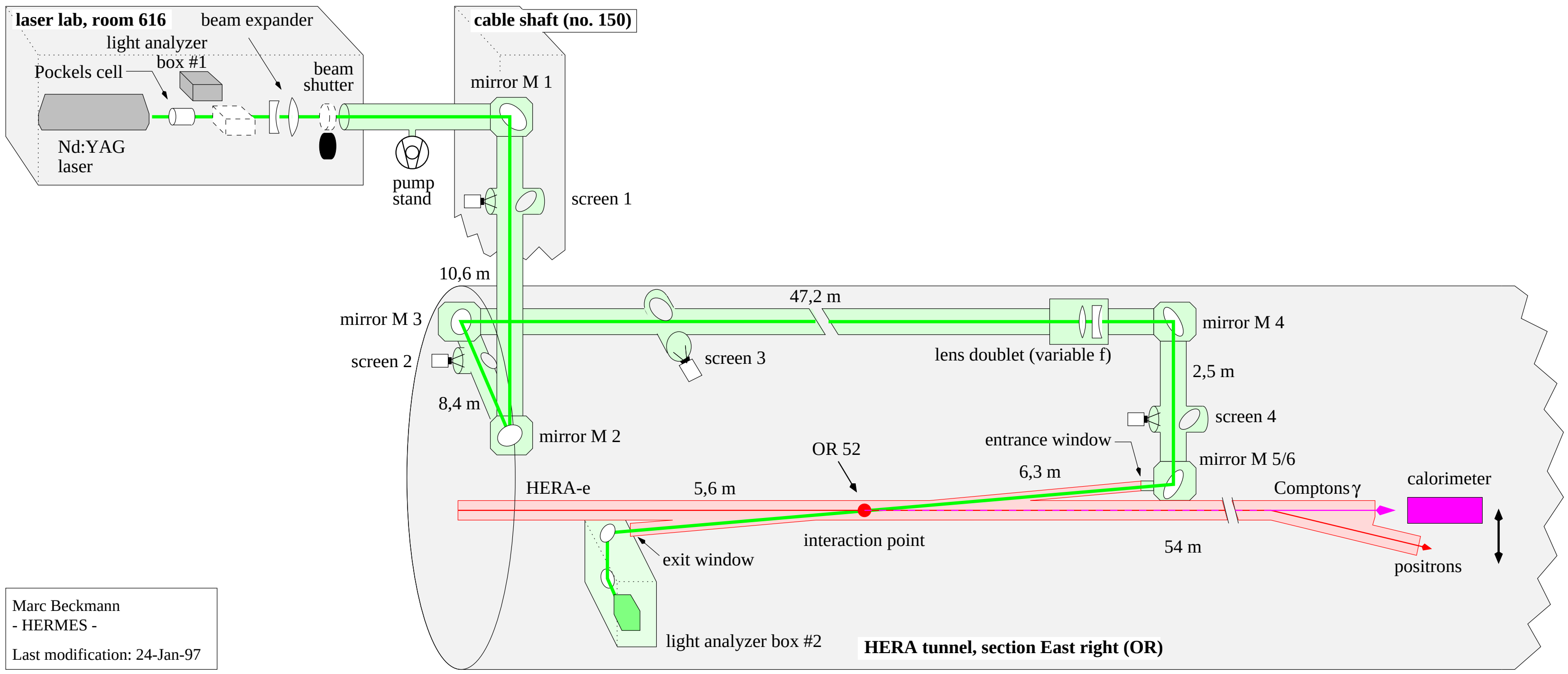

Figure 1: Layout of the Longitudinal Polarimeter.

3 Measurements

The measurement of the polarisation is performed in the analyser box 1 and 2. Each box consists of a motor controlled half-wave plate, a Glan-Thompson prism to select linear polarised light and a beam dump with a photo diode to measure the light intensity. The half-wave plate is typically rotated by 90 degrees in steps of 10 degrees. Figure 2 shows an example scan. The data points are fitted by the function:

$$I(\psi) = I_{\mathrm{mean}} + I_{\mathrm{Amp}} \sin(\psi \frac{4\pi}{180^\circ} - \alpha \frac{\pi}{180^\circ}), \tag{1}$$

where ψ is the rotation angle of the half-wave plate and $\alpha = 2\psi_{lin} - 90^\circ$ with ψ_{lin} the orientation of the ellipses or the linear light. The linear and circular polarisation S_1 and S_3 are calculated according to

$$S_1 = \frac{I_{\mathrm{Amp}}}{I_{\mathrm{mean}}} \tag{2}$$

and

$$S_3 = \sqrt{1 - S_1^2}, \tag{3}$$

respectively.

To measure the optical effects of the laser transport system we have used two types of polarised light. First we use linear polarised light as produced by the laser. Here we can rotate the direction using a half-wave plate from 0 to 90 degrees. Second we use elliptical (and ideally circular) polarised light as produced by the Pockels cell. At the Pockels cell we can adjust the positive and negative voltages to produce right and left polarised light. By varying the voltages the fraction of circular polarised light compared to linear polarised light is changed up to no circular light and 100% linear polarised light. In the first measurements we have only measured the polarisation at a certain set of voltages, where we would expect high circular polarisation. Later we have performed full high-voltage scans from 0 to 3000 V in steps of 100 V.

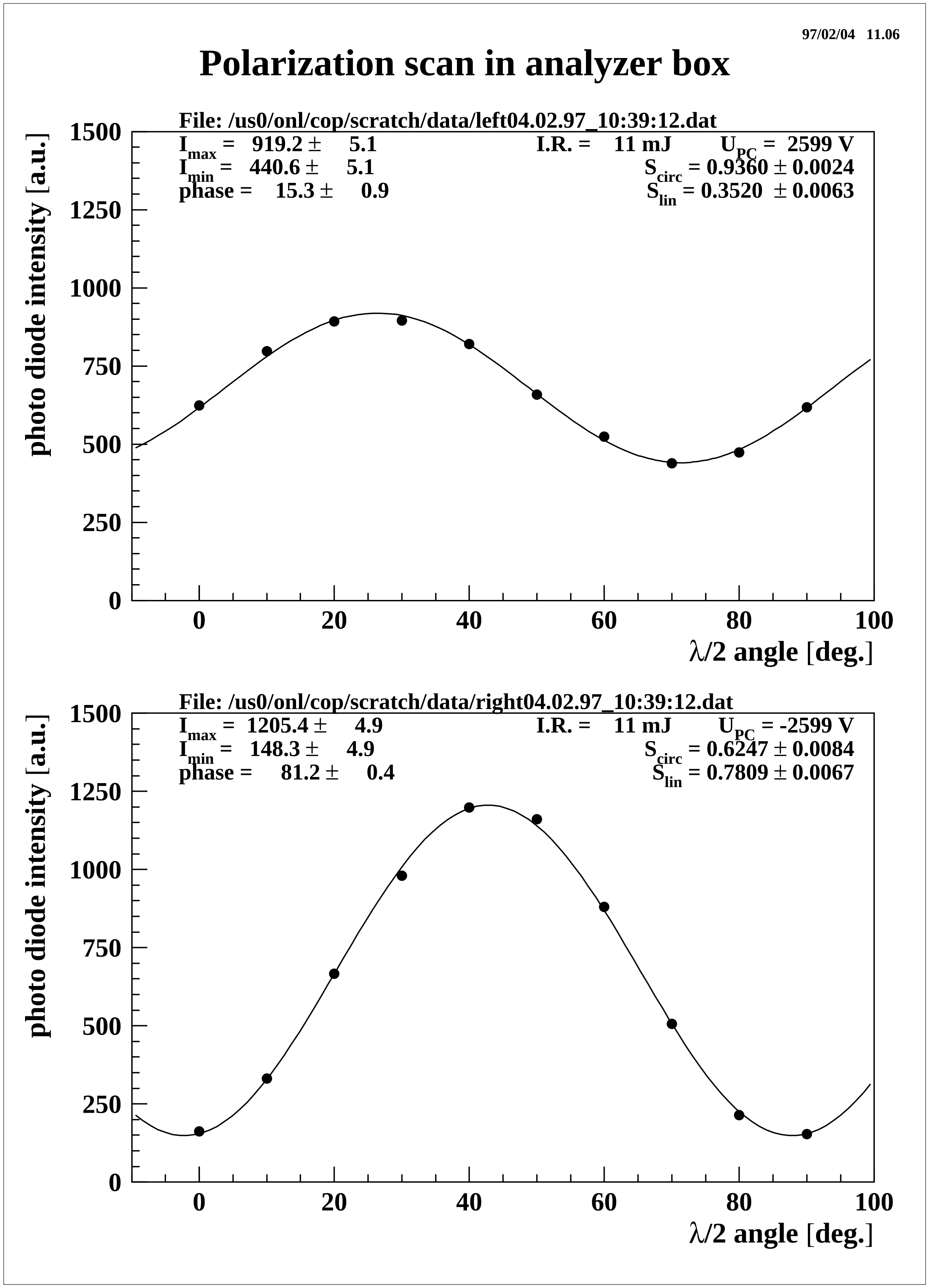

Figure 2: Polarisation scan in analyser box 2.

3.1 Measurements with linear polarised light

We have rotated the linear polarised light from the laser with a half-wave plate by an angle ψ. The half-wave plate was rotated between the angle $\psi/2 = 0°$ and $\psi/2 = 90°$ thus producing all orientations of linear light. The light transmitted through the optical system was measured in the analyser boxes. Here we measure the amount of linear polarisation and the angle of the linear light relative to one quantisation axis. This allows us to determine a possible shift of the phase of the light, which would reduce the relative amount of linear polarisation and a change in the orientation relative to the angle ψ of the incoming light. An example scan is shown in Fig. 3. These scans are fitted to the following functions:

$$\begin{aligned} S_1(\psi) &= S_1^{\text{mean}} + S_1^{\text{amp}} \sin(4\psi - \gamma), \\ \delta(\psi) &= \delta^{\text{mean}} + \delta^{\text{amp}} \cos(4\psi - \gamma). \end{aligned} \tag{4}$$

Here $S_1(\psi)$ is the measured linear polarisation, ψ is two times the angle of the half-wave plate. $\delta(\psi)$ is calculated from the fitted angle α (Eq. 1) of the analyser box measurement as follows:

$$\delta(\psi) = 0.5(\alpha(\psi) - \alpha(0)) - \psi.$$

The results of the fits are listed in table 2.

id	$S_1^{mean} - 100\%$	S_1^{amp}	δ^{mean}	δ^{amp}	γ
page	[percent]	[percent]	[degree]	[degree]	[degree]
A) 243	-1.50 ± 0.32	-3.28 ± 0.47	-0.49 ± 0.13	-1.13 ± 0.18	-66.2 ± 5.8
B) 248	-1.15 ± 0.28	-3.32 ± 0.39	0.51 ± 0.11	-0.91 ± 0.16	-66.9 ± 5.6
C) 252	-1.43 ± 0.28	-3.28 ± 0.40	-0.56 ± 0.12	-0.79 ± 0.16	-86.0 ± 5.9
D) 253	1.13 ± 0.29	1.85 ± 0.42	-0.63 ± 0.12	0.87 ± 0.16	75.7 ± 8.3
E) 254	1.87 ± 0.24	-0.27 ± 0.35	0.05 ± 0.12	-0.31 ± 0.16	1.1 ± 28.1
F) 256	0.46 ± 0.30	-1.19 ± 0.39	-0.24 ± 0.13	-0.34 ± 0.20	-85.2 ± 18.3
G) 258	-2.03 ± 0.27	3.90 ± 0.40	-0.56 ± 0.11	2.16 ± 0.15	26.3 ± 3.4
H) 259	-1.06 ± 0.32	2.73 ± 0.46	-1.41 ± 0.12	0.94 ± 0.17	38.7 ± 6.9
I) 276	-1.98 ± 0.23	4.16 ± 0.32	-0.98 ± 0.11	1.13 ± 0.16	66.1 ± 4.0

Table 2: Result of fits to linear scans in analyser box 2. The page numbers refer to the page in the LongPol logbook I.

The optical property of the windows, mirrors and lenses are approximated by the following matrix, which accounts for arbitrary changes of the phase ϕ (for the definitions of *rotate* and *PC* see Appendix A.4 and section 4):

$$\text{rotate}(\text{PC}(\phi), \beta),$$

where β is an rotation angle defined in transverse direction of the laser beam. The linear scans can be simulated by the following function:

$$\begin{aligned} S_1(\psi) &= S_1(\text{rotate}(\text{PC}(\phi), \psi)) \times (1, 0)) \\ \delta(\psi) &= \text{phase}(\text{rotate}(\text{PC}(\phi), \psi)) \times (1, 0)), \end{aligned} \tag{5}$$

where $S_1()$ calculates the amount of linear polarisation of the given vector and *phase* the difference of the phase between the two quantisation axes of the light. In Fig. 4 an example for $\phi = 0.4$ is plotted. The free parameter ϕ can be determined from both S_1 and δ:

$$\phi = \arccos(1 - 2S_1^{\mathrm{amp}})$$

$$\mathrm{phase}(\mathrm{rotate}(\mathrm{PC}(\phi), \pi/8)) \times (1,0)) = \delta^{\mathrm{amp}}$$

The values of ϕ are listed in table 4 on page 14.

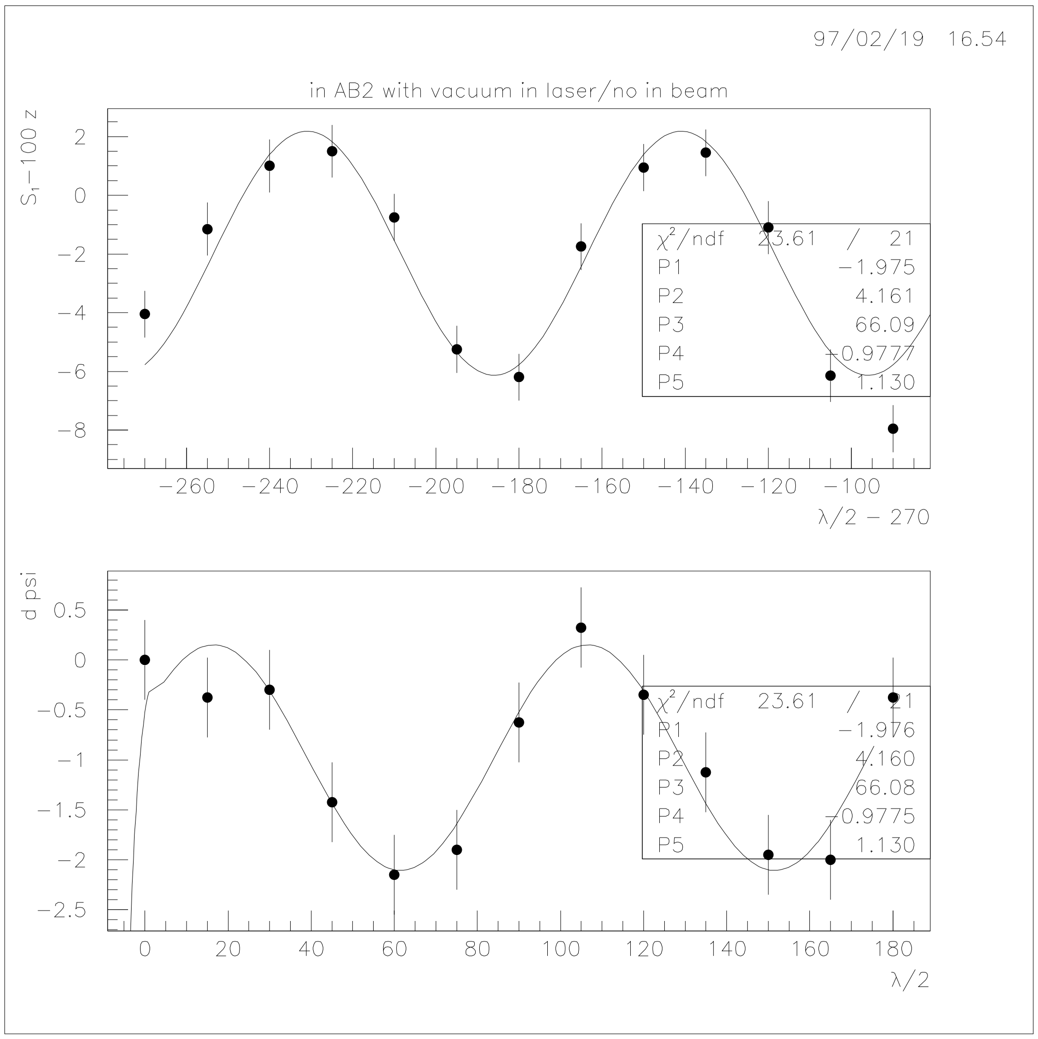

Figure 3: Linear scan in analyser box 2 (I 276). Units of $\lambda/2$ and *d psi* are degree, $S_1 - 100$ is measured in percent.

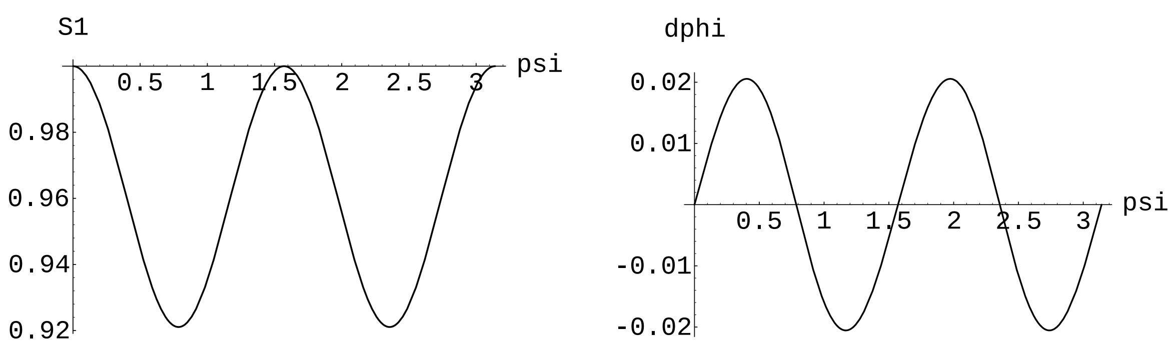

Figure 4: Simulation of linear scan in analyser box 2 for $\phi = 0.4$. S_1 is given in absolute values, $dphi$=$\delta(\psi)$ and psi in rad.

3.2 High-voltage scans

For the same experimental conditions where the linear scans were done the high voltage in the Pockels cell was scanned. The Pockels cell produces circular light at the voltage $V_{\lambda/4}$. In general the phase between the two quantisation axes of the light is proportional to the Pockels cell voltage U:

$$\phi = \frac{U}{V_{\lambda/4}}\frac{\pi}{2}.$$

Figure 5 shows an example scan of the high voltage U measured in AB2. The curve is fitted to the function

$$S_3 = S_{QW}\sin\left(\frac{U - V_{off}}{V_{QW} - V_{off}}\frac{\pi}{2}\right), \tag{6}$$

where S_{QW} is the maximum circular polarisation, V_{QW} the voltage at that point and V_{off} the voltage at $S_3 = 0$.

The circular polarisation can also be calculated by the Jones matrices as follows:

$$S_3(U) = S_3\left[rotate(PC(\phi), \beta) \times rotate(PC\left(\frac{U}{V_{\lambda/4}}\frac{\pi}{2}\right), \frac{\pi}{4}) \times (0,1)\right] \tag{7}$$

Here $rotate(PC(\phi), \beta)$ is the parametrisation of the optical elements between the Pockels cell and the analyser box, $V_{\lambda/4}$ the quarter wave voltage of the Pockels cell (which can be measured by a HV scan in AB1) and the functions S_3, $rotate$ and PC are defined in Appendix A.4. Figure 6 shows a simulated high-voltage scan for $\phi = 0.4$, $\beta = 1.98$ and $V_{\lambda/4} = 2045$ V.

The results of the fits to the high-voltage scans are summarised in table 3 and used in section 4 to determine ϕ and β using Eq. 7.

Id page AB	pos. voltage	neg. voltage	left circ. pol.	right circ. pol.	pos V_{off}	neg V_{off}	condition
246 1	2012 (2)	1936 (2)	0.9999 (1)	0.9972 (2)	62	-72	analyser box 1
A) 247 2	2290 (5)	1706 (4)	0.9442 (12)	0.9250 (13)	409	-146	
C) 251 2	2308 (4)	1694 (4)	0.9532 (10)	0.9342 (4)	435	-170	
D) 252 2	2205 (3)	1744 (3)	0.9731 (9)	0.9692 (3)	385	-34	
E) 254 2	2079 (2)	1949 (3)	0.9906 (5)	0.9830 (3)	258	120	
F) 255 2	2249 (3)	1769 (3)	0.9848 (6)	0.9848 (6)	385	-98	
G) 256 2	2529 (3)	1490 (3)	0.9988 (3)	0.9942 (4)	631	-396	
258 1	2045 (2)	1916 (2)	0.9994 (2)	0.9987 (2)	102	-103	analyser box 1
H) 258 2	2371 (1)	1550 (3)	0.9853 (3)	0.9773 (8)	544	-295	
270 2	2385 (3)	1612 (3)	0.9865 (5)	0.9820 (6)	485	-237	new entrance window
I) 276 2	2435 (4)	1574 (3)	0.9727 (6)	0.9612 (8)	524	-293	
J) 278 2	2367 (4)	1603 (3)	0.9596 (8)	0.9442 (9)	467	-225	
K) 279 2	2256 (2)	1676 (2)	0.9821 (5)	0.9713 (8)	396	-128	

Table 3: Result of the fits to the high voltage scans.

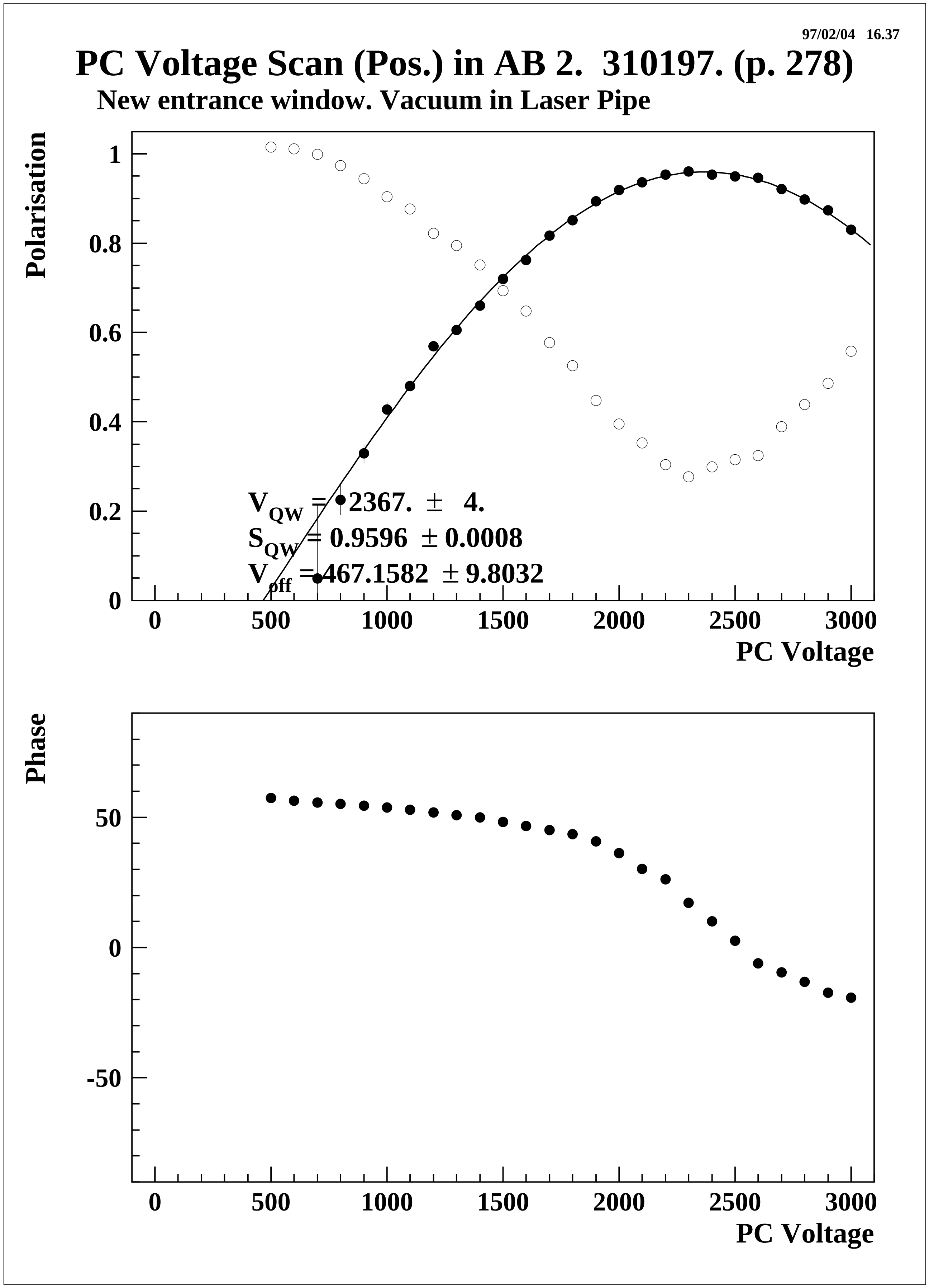

Figure 5: High voltage scan in analyser box 2.

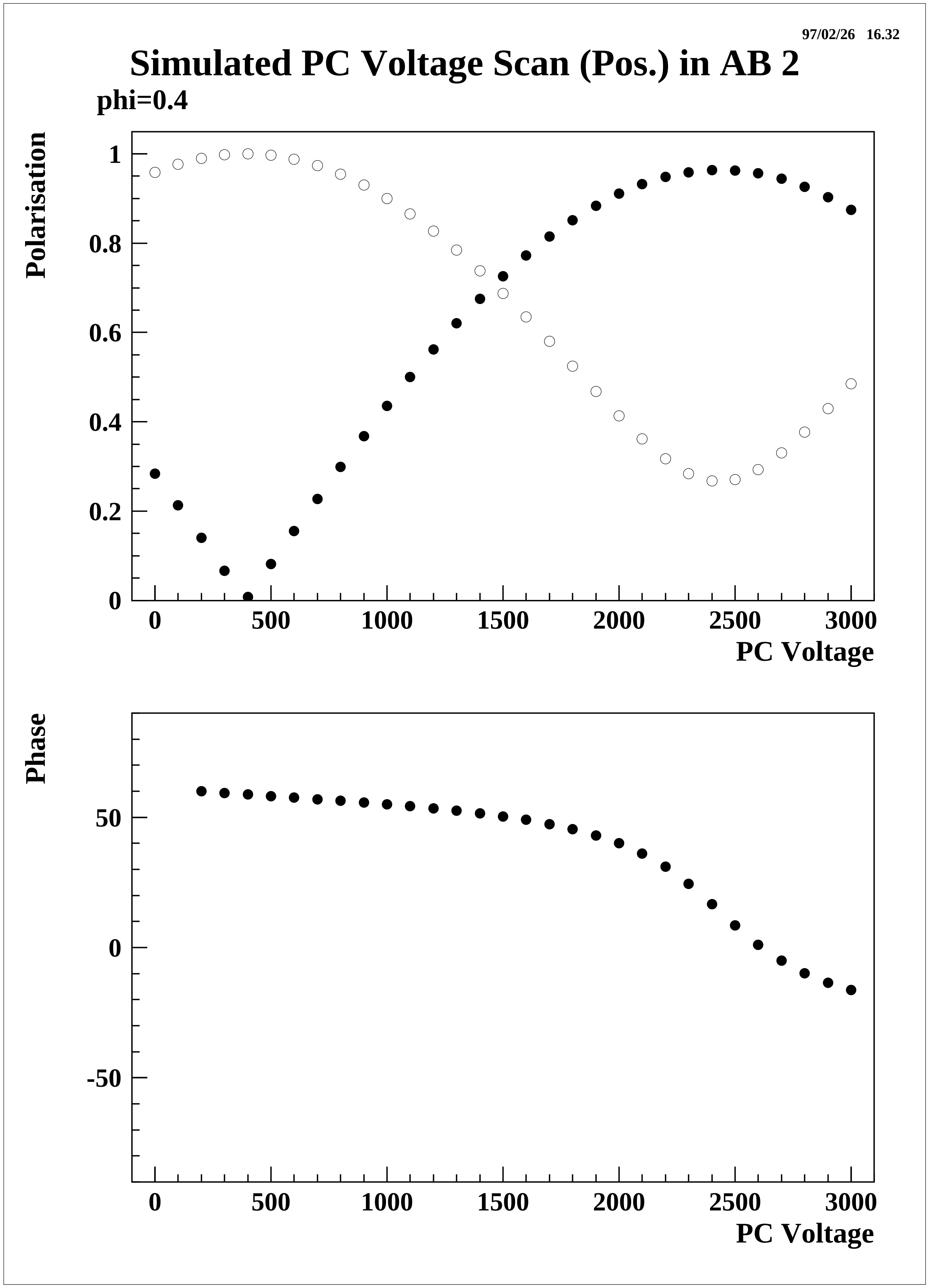

Figure 6: Simulated high voltage scan in analyser box 2.

3.3 Results of the circular polarisation measurements

Circular polarised light transmitted through optical elements, which cause shifts of the phase, is transformed into elliptical light. The shift ϕ of the phase can be determined by measuring the circular or linear light according to the equation:

$$\begin{aligned} \phi &= \arccos S_3 \\ \phi &= \arcsin S_1 \end{aligned} \tag{8}$$

Note that for $S_1 \ll 1$ $\phi \approx S_1$.

By comparing the measurement of the polarisation with and without optical elements we can determine the phase shift of the various components. The measurement error of S_1 is typically around 0.05, hence the error in ϕ is about the same.

The results are:

Old window 1: Mounted window 1, measured before M5/6 and in AB2: $\phi = 0.24 \pm 0.05$ and $\phi = 0.30 \pm 0.06$, respectively. Measured without mount in AB1: $\phi = 0.07 \pm 0.04$. Remounted, but screws not fixed, measured before M5/6: $\phi = 0.11 \pm 0.04$. Fully mounted window 1 measured before M5/6: $\phi = 0.06 \pm 0.05$ and at AB2: $\phi = 0.12 \pm 0.07$

Mirrors and lenses: Mirrors 1 to 4, lens dublett and beam expander measured before M5/6: $\phi = -0.02 \pm 0.04$. This value can be compared to the measurement of two phase compensated mirrors of $\phi = 0.025 \pm 0.011$. This value is calculated from the circular polarisation with ($S_3 = 0.9974 \pm 0.0006$) and without ($S_3 = 0.9989 \pm 0.0003$) two mirrors [2].

Steering of the beam: From three measurements in AB2 of 0.36 ± 0.09, 0.42 ± 0.07 and 0.38 ± 0.07 one can estimate that the effect of the steering is smaller than the measurement error of $\sigma_\phi = 0.05$.

HERA vacuum: Filling of the HERA beam pipe with N_2 yields $\phi = 0.01 \pm 0.05$.

Laser pipe: Vacuum in the laser beam pipe with N_2 in the HERA beam pipe gives a change of $\phi = 0.06 \pm 0.05$.

Window 2: Monitoring the effect of fixing the screws on the HERA entrance beam window gives a value of $\phi = 0.09$.

Window 3: The same effect on the HERA beam exit window gives a value of $\phi = 0.13$. Here the screws had to be fixed a little bit more after this measurement. So we expect the final value of phi to be different.

ϕ_2	ϕ_3	comment
0.15	0.22	screws loose
0.23	0.30	1/8 turn
0.23	0.37	3/16 turn
0.24	0.35	1/4 turn (fixed)

Window 2 and 3: The measurement of the HERA window with loose screws does not include the effect of the windows itself and mirrors 7 and 8. From the measurement in AB2 with no window 1, air in the laser pipe and N_2 in the HERA beam pipe one can estimate an upper limit of $\phi = 0.15$ for both windows together.

4 Results

The effect of the windows and the other optical elements are modelled by the following Jones Matrix:

$$\text{rotate}(\text{PC}(\phi), \beta) = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \times \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \tag{9}$$

This matrix describes arbitrary polarised light. The angle ϕ gives the shift of the phase in one direction and the angle β defines the rotation angle of the element.

Multiple optical elements can be described by a matrix multiplication of the corresponding matrices. In the case of circular polarised light the following equation holds for the phases ϕ of the elements 1 and 2:

$$|\phi_1 - \phi_2| \leq \phi_{1+2} \leq \phi_1 + \phi_2, \tag{10}$$

where the equal sign is true for $\beta_1 = \beta_2$ and $\phi_{1+2} = 0$ for $\phi_1 = \phi_2$ and $\beta_1 = \beta_2 \pm \pi/2$ (all ϕ positive).

Table 4 lists the values for ϕ and β of the total system determined from the high-voltage scans and the linear scans. When determining the angle β one has an ambiguity of $\pi/2$ from the left/right circular light and one ambiguity due to the unknown orientation of the fast and slow axes of the Pockels cell.

id	W1	laser	W2	HERA	W3	ϕ^{HV}	ϕ^{lin}	β
A)	old	vac.	fixed	vac.	fixed	0.41 ± 0.01	0.38 ± 0.02	0.17,1.4/ 1.7,2.97±0.1
B)	old	vac.	fixed	N_2	fixed		0.36 ± 0.01	
C)	old	air	fixed	N_2	fixed	0.39 ± 0.01	0.36 ± 0.02	0.2,1.3/1.6,3.0
D)	old	air	loose	N_2	loose	0.28 ± 0.01	0.31 ± 0.04	0.33,1.22/1.92,2.78
E)		air	loose	N_2	loose	0.16 ± 0.02	0.16 ± 0.05	1.9
F)		air	fixed	N_2	loose	0.23 ± 0.01	0.22	1.9
G)		air	fixed	N_2	tight.	0.39 ± 0.01	0.47 ± 0.07	0.75/2.35
H)		air	fixed	N_2	fixed	0.35 ± 0.02	0.34 ± 0.01	2.0
I)	new	vac.	fixed	vac.	fixed	0.40 ± 0.01	0.40 ± 0.01	1.98
J)	new	vac.	fixed	vac.	fixed	0.41 ± 0.01		
K)	new	air	fixed	vac.	fixed	0.29 ± 0.02		

Table 4: Fitted parameters ϕ and β of the model

By comparing the measurements, where only one condition was changed, one can calculate the contribution to the total phase ϕ due to that change. Table 5 lists these values, determined from table 4 and the circular polarisation measurements (section 3.3). Here window *fixed* means the the screws of the windows were tightened. Windows 2 and 3 (the HERA entrance and exit window) could not be measured independently because it was not allowed to dismount them. There we have only an upper limit for the contribution of one of these windows to the total phase change of ($\phi = 0.16$). During these measurements we have exchanged window one by a less birefringent window. This reduced the phase change by $\phi = 0.06$.

Table 6 finally lists the estimated phase changes for the three windows. From this we can estimate the phase change at the IP. From the phase we can calculate the values of the linear polarisation S_1 and circular polarisation S_3 under assumption of circular polarised light from

condition	ϕ from HV/linear scan	ϕ from circ. meas.
Old window 1	0.13±0.03	0.06±0.05
		0.12±0.07
HERA vacuum	0.02±0.02	0.01±0.05
Laser pipe vacuum:		
(Hera N_2)	0.00±0.02	0.06±0.05
(Hera vacuum)	0.12±0.02	
Window 2 (fixed)	0.07±0.02	0.09±0.05
Window 3 (fixed)	0.12±0.02	
Window 1+2 (not fixed)	$<$ 0.16±0.02	0.15±0.04
New window 1	0.06±0.02	

Table 5: Summary of all results for parameter ϕ of the model

the Pockels cell (before window 1). The estimated range of phase change at the IP of ϕ =0.13–0.28 corresponds to circular polarisations of 0.992–0.96. We convert this range to a value of $98 \pm 2\%$ circular polarisation at the interaction point. Of course this value can be increased by producing elliptical polarised light, so that the effects of the windows are cancelled (see Appendix A.3).

Position	ϕ	S_1	S_3
Window 1	0.06±0.02	0.06±0.02	0.998
mirrors	0.00±0.04	0.00±0.04	1.000
Window 2	0.07–0.22	0.07–0.22	0.997–0.975
Window 3	0.12–0.27	0.12–0.27	0.993–0.963
At IP	0.13–0.28	0.13–0.28	0.992–0.96
In AB 2	0.40±0.01	0.41±0.01	0.92±0.01

Table 6: Phase shifts, linear polarisation and circular polarisation for incoming circular light.

5 Conclusion

From the measurements at the Longitudinal Polarimeter under different operating conditions one can draw the following conclusions:

- The mirrors and lenses preserve circular polarisation.
- The three windows in operation are birefringent and cause phase shifts between 0.06 and 0.27 rad.
- The mounting of the windows is the most important contribution. The mounting method alone causes up to 0.12 rad in the phase shift. The vacuum pressure on the window can cause additional effects, but these are smaller.
- The total phase shift of the laser transport system to the interaction point is between 0.13 and 0.28 rad. This corresponds to a shift of $(0.021 - 0.045)\ \lambda$.

With the current setup we reach $98 \pm 2\%$ circular polarisation at the interaction point and $92 \pm 0.01\%$ circular polarisation in the analyser box 2, if we start with circular polarised light at the Pockels cell. These values can be improved by using different high voltages to produce left and right circular light or by using a second Pockels cell.

To reach a level of less than 10% linear polarisation, i.e. more than 99.5% circular polarisation one needs to modify the current window mounting. Each window should produce less than 3% linear light in case of incoming circular light.

A Appendix

A.1 How to adjust the Pockels Cell

To exchange the Pockels cell dismount it from the rotator by unscrewing the one screw at the top in the Al-ring. Afterwards unscrew the Al-ring from the rotator and mount the new one for the other Pockels cell. Note that the mark on the Al-ring should be around 55 and 75° (when looking in the beam direction on the left the side, i.e. for negative values). For the white PC align the mark on the PC with the mark on the Al-ring. For the black PC align the HV-connectors with the screw on the Al-ring.

Adjustment off x-y position

Use 1-2 mJ beam to adjust roughly the x-y position.

Rotation in x-z and y-z Plane

Switch PC high-voltage off and note beam position after PC on beam-spot screen. Depolarise laser beam in front of PC by using *Tesafilm* (scotch tape). Mount Glan-Thompson prism behind the Pockels cell. Center the pattern on the beam-spot screen around the previously noted beam position by rotating the table in x-z and y-z direction. There is a screw to fix the horizontal position below the rotator. For more information see the documentation of the Pockels cell.

Once the PC is adjusted also by rotating (see next paragraph) one can do a fine scan. For this adjust the PC HV to the mean maximum value as found by the HV scan. Then vary first the x-z and then the y-z plane in steps of 0.01 (equal or less than one unit of the scale!). Optimise the linear polarisation for the one HV setting. Optimal values should be below 4% linear polarisation. There should be no left right asymmetry.

Rotation in x-y Plane

The circular polarisation is optimised by rotating the PC. For the white PC the mark should be near -71° for the black PC close to 60°. After roughly adjusting do one HV scan (from 400 to 3000 V in steps of 200 V is sufficient). Then rotate the PC by the following angle according to the equation:

$$\Delta\phi[^\circ] = \arccos(S_3)\frac{90^\circ}{\pi}$$

One has to rotate in either positive or negative direction. The correct value should be reached within two iterations.

For a given angle $\Delta\phi$ one expects the following amount of linear polarisation:

$$S_1 = \sin(\Delta\phi\frac{0.5\pi}{180^\circ})$$

Calibration values of the Pockels cell

Currently the Pockels cells are well adjusted by using the values given in the following table. These values can be used as start values for adjustments.

	black PC	*white PC*
x-z	0.86	0.83
y-z	2.74	2.75
x		
y		
x-y	59°	71°
U_+	2120 V	2000 V
U_-	2120 V	2000 V

A.2 Calibration of high voltage power supply for the Pockels cell

The high voltage was calibrated by measuring the positive and negative high voltage independent by using a HV probe (1:1000) and a volt-meter. Throughout this note all voltages reflect the uncalibrated values as set within the cop program.

Cop	HV-Input	HV
+2000	1.900	1864
-2000	-1.875	1874
+1800	1.712	1679
-1800	-1.685	1686

$$\text{Pos. Voltage} = \text{cop-value} * 0.932$$

$$\text{Neg. Voltage} = \text{cop-value} * 0.937$$

A.3 Operation with two Pockels cells

Two Pockels cells (PC) can be used to compensate for any phase shifts in the windows or elsewhere. It is possible to produce arbitrary elliptical light. The first PC has to be mounted in standard mode, i.e. rotated by 45° from the polarising axis. The second Pockels cell needs to be mounted behind the first one with the polarising axis at 0° i.e. rotated by $-45°$ compared to the first PC.

Let U_1 and U_2 be the high voltage for the first and second PC, respectively. Then the phase differences produced by the PCs defined by:

$$\phi_1 = \frac{U_1}{U_1^{\lambda/4}}\frac{\pi}{2} \tag{11}$$

$$\phi_2 = \frac{U_2}{U_2^{\lambda/4}}\frac{\pi}{2} \tag{12}$$

The circular polarisation is:

$$S_3 = \cos(\pm\phi_1)\sin(\pm\phi_2) \tag{13}$$

From this equation one concludes that constant values of S_1 and hence S_3 can be reached for values $\phi \approx S_1, \phi \ll 1$ by the voltage pair of

$$\phi_1 = \pm\phi\sin(\beta) \tag{14}$$

$$\phi_2 = \mp(\phi\cos(\beta) + \frac{\pi}{2}) \tag{15}$$

Here β is the orientation of the generated ellipses.

For a known circular polarisation or phase shift ϕ one should scan the voltages by varying β in the above equation until one reaches the maximum circular polarisation. See picture 7.

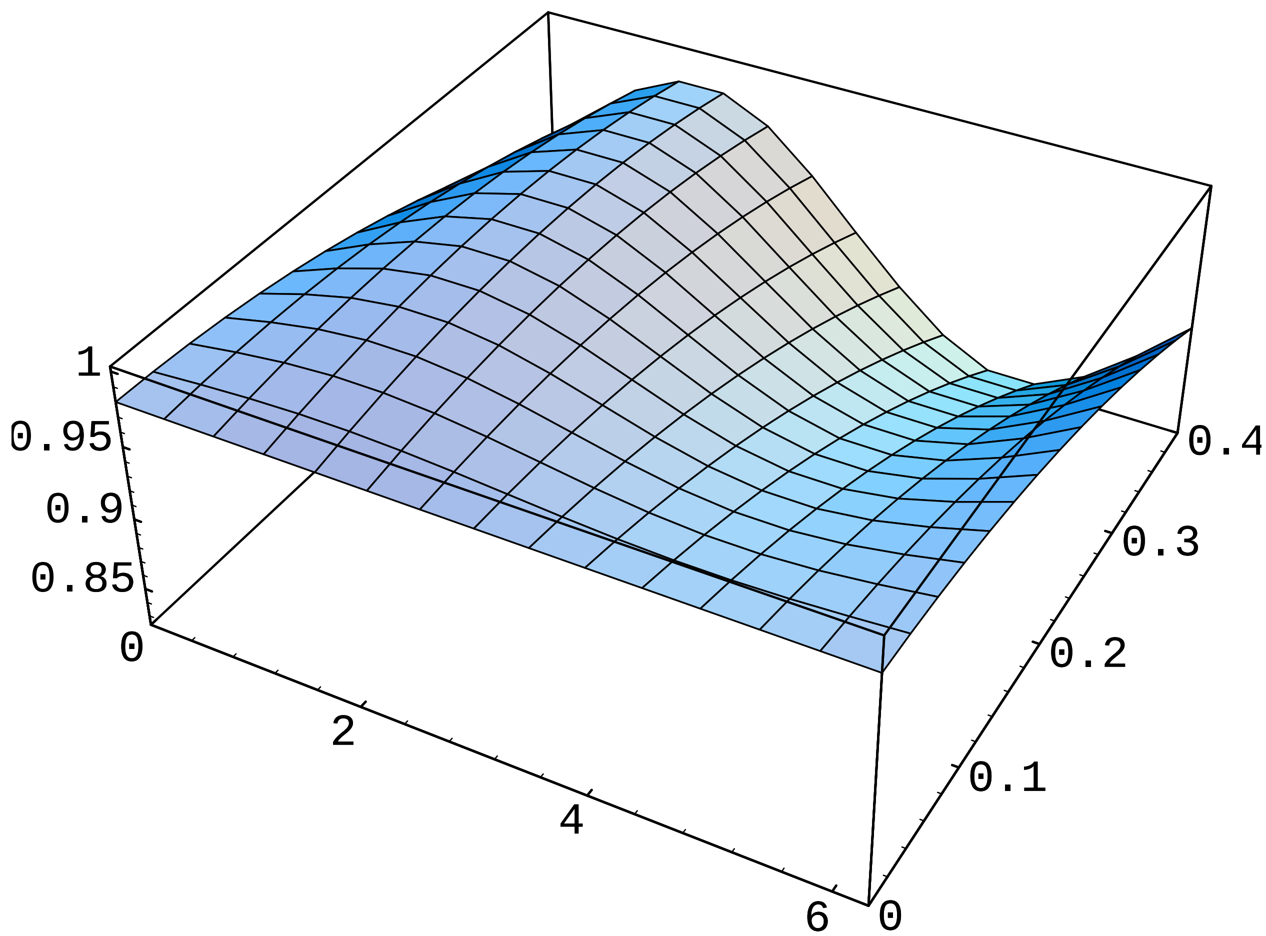

Figure 7: Circular polarisation for different high voltage combinations. Plotted are parameter $\phi = 0 - 0.4$ versus $\beta = 0 - 2\pi$.

A.4 Theoretical description of polarised light

Most of these equations are taken from Refs. [3, 4].

Fraction of circular polarisation of light $V = \binom{E_1 e^{\phi_1}}{E_2 e^{\phi_2}}$:

$$S_3 \begin{pmatrix} E_1 e^{\phi_1} \\ E_2 e^{\phi_2} \end{pmatrix} = 2E_1 E_2 \sin(\phi_1 - \phi_2) \tag{16}$$

Fraction of linear light polarisation:

$$S_1(V) = \sqrt{1 - S_3(V)} \tag{17}$$

Orientation of elliptical and linear light:

$$\text{phase} \begin{pmatrix} E_1 e^{\phi_1} \\ E_2 e^{\phi_2} \end{pmatrix} = \frac{1}{2} \arctan\left(\frac{2E_1 E_2 \cos(\phi_1 - \phi_2)}{E_1^2 - E_2^2}\right) \tag{18}$$

Jones matrices for optical elements:
Pockels cell at voltage U and quarter-wave voltage $U_{\lambda/4}$:

$$\mathrm{PC}(U) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{U}{U_{\lambda/4}}\frac{\pi}{2}} \end{pmatrix} \tag{19}$$

Glan-Thompson Prism:

$$\mathrm{GTP} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{20}$$

Half-wave plate:

$$\mathrm{HWP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{21}$$

Rotation of an optical element M by α:

$$\mathrm{rotate}(\alpha, M) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \times M \times \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{22}$$

A.5 Definitions for Mathematica

Most of the calculations were done using Mathematica [5]. In this section the most important equations are listed.

```
(* general definitions *)

rotate[phi_] := {{Cos[phi],-Sin[phi]},{Sin[phi],Cos[phi]}}
rot[m_,phi_] := rotate[phi] . m . Transpose[rotate[phi]]

(* 2 Ex Ey sin delta *)
circ[vector_] := Sin[Arg[vector] . {{1},{-1}}]
                 (Abs[vector] . Reverse[Abs[vector]])
linear[vector_] := Sqrt[1-circ[vector]^2]

phi[vector_]:=ArcCos[circ[vector]]

(* tan (2 phase) = 2 E1 E2 cos delta/(E1^2-E2^2) *)
phase[vector_] := 0.5 ArcTan[
      (Abs[vector] . {{1,0},{0,-1}} ). Abs[vector],
      Cos[Arg[vector] . {{1},{-1}}] (Abs[vector] . Reverse[Abs[vector]])]

pos[x_] := If[x<0,x+Pi,x]

(* normalised to intensity of 1 *)
norm[vector_] := vector / Sqrt[Abs[vector] . Abs[vector]]

(* Jones matrices for optical elements  *)
(* Pockels cell, Glan-Thompson Prism, Half Wave plate *)
```

```
PC[phi_] := {{1,0},{0,Exp[I phi]}}
GTP = {{1,0},{0,0}}
HWP = {{1,0},{0,-1}}

(* Analyser box *)
A = GTP . rot[HWP,phase]

(* Intensity of photo diode *)
PD[m_] := m . Conjugate[m]

(* plot of scan in analyser box *)
g1=Plot[PD[A . rot[PC[-Pi/2],Pi/4+0.1] . {1,0}],{phase,0,Pi/2}]

(* Generic analysis of linear scan *)
p1[lin_]:=ArcCos[1-0.02 lin]
p2[psi_,start_]:=FindRoot[180/Pi phase[rot[PC[phi],Pi/8]
. {1,0}][[1]]==psi,{phi,start,start+0.01}]

a1[v1_,v2_]:=((v1-2045)/2045 -(v2-1916)/1916)/2 Pi/2

g1[v1_,v2_]:=Plot[{Abs[circ[rot[PC[p],phi] . rot[ PC[Pi/2+a1[v1,v2]],Pi/4 ]
. {0,1} ]][[1]],
Abs[circ[rot[PC[p],phi] . rot[ PC[Pi/2-a1[v1,v2]],Pi/4 ] . {0,1} ]][[1]]},
{phi,0,Pi}]


(* simulate HV scan results *)
Plot[phase[rot[PC[0.4],1.98] .  rot[ PC[U/2045 Pi/2],Pi/4 ] .
               {0,1}][[1]],{U,0,3000}]
Plot[circ[rot[PC[0.4],1.98] .  rot[ PC[U/2045 Pi/2],Pi/4 ] .
              {0,1} ][[1]],{U,0,3000}]
Plot[linear[rot[PC[0.4],1.98] .  rot[ PC[U/2045 Pi/2],Pi/4 ] .
            {0,1} ][[1]],{U,0,3000}]
Plot[Abs[circ[rot[PC[0.4],1.98] .  rot[ PC[-U/1916 Pi/2],Pi/4 ] .
         {0,1} ]][[1]],{U,0,3000}]

Do[WriteString["circ.txt",U," ",
N[Abs[circ[rot[PC[0.4],1.98] .  rot[ PC[-U/1916 Pi/2],Pi/4 ] . {0,1} ]][[1]]]
,"\n"],{U,0,3000,100}]
Do[WriteString["circ.txt",U," ",
N[Abs[circ[rot[PC[0.4],1.98] .  rot[ PC[U/2045 Pi/2],Pi/4 ] . {0,1} ]][[1]]]
,"\n"],{U,0,3000,100}]

!!circ.txt

////// windows (D)
p1[1.85]
p2[0.87,%]
```

```
p=%%
p = (0.27 + 0.35)/2
g1[2205,1744]
//a= 0.33,1.22 / 1.92, 2.78
(* Model for IP *)
Plot[{Abs[circ[rot[PC[0.29],2.0] . rot[ PC[U/2045 Pi/2],Pi/4 ] . {0,1} ]][[1]],
    Abs[circ[rot[PC[0.29],2.0] . rot[ PC[-U/1916 Pi/2],Pi/4 ] . {0,1} ]][[1]]},
{U,0,3000}]
Plot[{Abs[circ[rot[PC[0.2],2.0] . rot[ PC[U/2045 Pi/2],Pi/4 ] . {0,1} ]][[1]],
    Abs[circ[rot[PC[0.2],2.0] . rot[ PC[-U/1916 Pi/2],Pi/4 ] . {0,1} ]][[1]]},
{U,0,3000}]
c1[v1_]:=N[Abs[circ[rot[PC[0.4],2.0] . rot[ PC[v1/2045 Pi/2],Pi/4 ] . {0,1} ]][[1]]]
c2[v1_]:=N[Abs[circ[rot[PC[0.4],2.0] . rot[ PC[-v1/1916 Pi/2],Pi/4 ] . {0,1} ]][[1]]]
FindMinimum[-Abs[circ[rot[PC[0.2],2.0] . rot[ PC[-U/2045 Pi/2],Pi/4 ] . {0,1} ][[1]]],{U,{2300,2330}}]
FindMinimum[-Abs[circ[rot[PC[0.13],2.0] . rot[ PC[-U/1916 Pi/2],Pi/4 ] . {0,1} ][[1]]],{U,{2300,2330}}]
```

References

[1] F. Burkart, *Messung der Stokes-Parameter am Laserstrahl des HERA-Polarimeters*, diploma thesis, Universität Freiburg, 1996.

[2] M. Beckmann, *Test of the new 37-layer CVI mirrors for the Longitudinal Polarimeter,* IPR-97-01, Jan 1997.

[3] E. Hecht, *Optik*, Addison-Wesley, Bonn, 1989.

[4] W. A. Shurcliff and S.S. Ballard, *Polarized Light*, D. van Nostrand Company, Inc., Princeton, NJ, 1964.

[5] S. Wolfram, *Mathematica – ein System für Mathematik auf dem Computer*, 2nd. Ed., Addison-Wesley Verlag, 1994.