March 28, 2007

Physics 390: Homework set #6
Solutions

Reading: Tipler & Llewellyn, Chapter 11, 12

Questions:

1. In many fission reactors, a large number of antineutrinos are emitted. Why?

Solution: Antineutrinos typically accompany beta-decay, when ${}^{A}_{Z}X \to {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$. The by-products of fission reactions often undergo beta decay, for the following reason. Fission reactors produce energy by splitting heavy nuclei, such as uranium or plutonium, into lighter nuclei with roughly half the mass of the parent. Because the parent nuclei contain significantly more neutrons than protons, their daughter nuclei are on the neutron-rich side of the "line of stability". These nuclei can move toward a more stable configuration by reducing their number of neutrons through the beta decay process $n \to p + e^- + \bar{\nu}_e$. Note that *free* neutrons that accompany fission reactions generally do *not* undergo beta decay. Long before their ∼10-minute half-life passes, they are "used up" in sustaining additional fission reactions.
Breeder reactors, though less widely used, produce additional antineutrinos through the chain ${}^{238}\mathrm{U} + n \to {}^{239}\mathrm{U} \to {}^{239}\mathrm{Np} + e^- + \bar{\nu}_e$ followed by the additional beta decay ${}^{239}\mathrm{Np} \to {}^{239}\mathrm{Pu} + e^- + \bar{\nu}_e$.
The fact that fission reactors produce a copious supply of electron antineutrinos is being exploited by several experiments that are investigating properties of neutrinos, including the very interesting possibility that they can "oscillate" into neutrinos of a different lepton species. The most notable of these experiments is the KamLAND experiment in Japan, a large underground detector that can detect neutrinos from a variety of nuclear power plants. KamLAND has shown that neutrinos indeed oscillate into different species. This proves that neutrinos have non-zero rest mass. (Japan is a particularly good location for this experiment since 1/3 of its power is produced at nuclear plants.)

2. Suppose we wish to do radioactive dating of a sample whose age we guess to be about t. Should we choose an isotope whose half-life is (a) $\gg t$; (b) $\sim t$; or (c) $\ll t$?

Solution: You should use an isotope with $t_{1/2} \sim t$. If $t_{1/2} \ll t$, then many half-lives will have passed and too much of the isotope will have decayed. If $t_{1/2} \gg t$, nearly all of the original amount will remain and it will be difficult to determine the age with precision. This is the reason why ${}^{14}\mathrm{C}$, which has a half-life of about 5700 years, is most useful for dating artifacts with ages ranging from about 500 to 50,000 years.

3. Many households in Michigan have elevated levels of radon gas in their basements. Where does this gas come from? Why does it accumulate in basements? How is it detected? Why is it considered a health hazard?

Solution: Radon is produced as part of the alpha-decay chain of naturally-occurring heavy elements such as uranium and thorium (see, e.g., Figure 11-18 on p. 528). This gas, the heaviest noble gas, accumulates in basements because it is heavier than air. Also, the ventilation in most basements is relatively poor, especially in wintertime. Radon is an alpha-emitter, with a half-life of about 3.8 days. This relatively short half-life makes it especially dangerous, since if it makes it into your basement there is a good chance it will decay there. Being a gas, radon can be inhaled, where the highly-ionizing alpha radiation can cause biological damage. In particular, it increases the risk of lung cancer. Radon can be detected using small ionization counters. The remedy for basements with elevated radon levels is usually to install additional ventilation to improve the circulation of fresh air.

Problems: Chapter 11: 9, 19, 28, 37
Chapter 12: 3, 8, 19

Problem 11-9: In general, the binding energy is given by Eqn. 11-11:

$$B = ZM_Hc^2 + Nm_Nc^2 - M_Ac^2.$$

(a) For ^{9}Be, with $Z = 4$ and $N = 5$:

$$\begin{aligned} B &= 4(1.007825uc^2) + 5(1.008665uc^2) - 9.012174uc^2 \\ &= 0.062451uc^2 = (0.0062451uc^2)(931.5 \text{ MeV}/uc^2) \\ &= \underline{58.2 \text{ MeV.}} \end{aligned}$$

Then

$$B/A = (58.2 \text{ MeV})/(9 \text{ nucleons}) = \underline{6.46 \text{ MeV/nucleon}}$$

(b) For ^{13}C, with $Z = 6$ and $N = 7$:

$$\begin{aligned} B &= 6(1.007825uc^2) + 7(1.008665uc^2) - 13.003355uc^2 \\ &= 0.104250uc^2 = (0.104250uc^2)(931.5 \text{ MeV}/uc^2) \\ &= \underline{97.1 \text{ MeV.}} \end{aligned}$$

Then

$$B/A = (97.1 \text{ MeV})/(13 \text{ nucleons}) = \underline{7.47 \text{ MeV/nucleon}}$$

(c) For ^{57}Fe, with $Z = 26$ and $N = 31$:

$$\begin{aligned} B &= 26(1.007825uc^2) + 31(1.008665uc^2) - 56.935396uc^2 \\ &= 0.536669uc^2 = (0.536669uc^2)(931.5 \text{ MeV}/uc^2) \\ &= \underline{499.9 \text{ MeV.}} \end{aligned}$$

Then

$$B/A = (499.9 \text{ MeV})/(57 \text{ nucleons}) = \underline{8.77 \text{ MeV/nucleon}}$$

Problem 11-19:

(a) From Eqn. 11-19,

$$R = -\frac{dN}{dt} = R_0 e^{-\lambda t}.$$

We know that at $t = 0$, $R_0 = 8000$ counts/s, and that at $t = 10$ min $R = 1000$ counts/s. So

$$1000 = 8000 e^{-\lambda t} \quad \Rightarrow \quad \frac{1}{8} = e^{-\lambda t} \quad \Rightarrow \quad \ln(1/8) = -\lambda t \quad \Rightarrow \quad \lambda = -\ln(1/8)/t.$$

Thus

$$\lambda = -\ln(1/8)/(10 \text{ min}) = 0.208 \text{ min}^{-1}.$$

Then the half-life, from Eqn. 11-22, is

$$t_{1/2} = \frac{\ln 2}{\lambda} = \underline{3.33 \text{ min}} = \underline{200 \text{ s.}}$$

(b) The decay constant is $\lambda = 0.208 \text{ min}^{-1} = \underline{3.47 \times 10^{-3} \text{s}^{-1}}$.

(c) At $t = 1$ min,

$$R = R_0 e^{-\lambda t} = (8000/\text{sec}) e^{-0.208 \cdot 1} = \underline{6500/\text{sec.}}$$

Problem 11-28: The electron capture reaction for ^{7}Be is

$$^7_4\text{Be}_3 \quad \rightarrow \quad ^7_3\text{Li}_4 + \nu_\text{e}.$$

(a) Yes, the characteristics of the decay would be altered. This is because under very high pressure the electrons are "squeezed" closer to the nucleus. The probability density of the electrons, particularly the K electrons, is increased near the nucleus making electron capture more likely, thus decreasing the half-life.

(b) Yes, the decay would be altered. Stripping all four electrons from the atom renders electron capture impossible, lengthening the half-life to infinity.

Problem 11-37: The range of a force mediated by an exchange particle of mass m is (Eqn. 11-50)

$$R = \frac{\hbar c}{mc^2},$$

so

$$mc^2 = \frac{\hbar c}{R} = \frac{197.3 \text{ MeV} \cdot \text{fm}}{0.25 \text{ fm}} = \underline{789 \text{ MeV.}}$$

This is very close to the mass of the ρ (rho) meson, 770 MeV.

Problem 12-3: The reaction of deuterium plus nitrogen-14 is

$$^{14}\text{N} + {}^{2}\text{H} \quad \rightarrow \quad {}^{16}\text{O}.$$

The possible products of this reaction are

$$\begin{aligned}
^{16}\text{O} &\rightarrow {}^{14}\text{N} + {}^{2}\text{H} \\
^{16}\text{O} &\rightarrow {}^{16}\text{O} + \gamma \\
^{16}\text{O} &\rightarrow {}^{15}\text{O} + \text{n} \\
^{16}\text{O} &\rightarrow {}^{15}\text{N} + \text{p} \\
^{16}\text{O} &\rightarrow {}^{12}\text{C} + \alpha
\end{aligned}$$

Problem 12-8: For the reaction $n + p \quad \rightarrow \quad d + \gamma$, the Q-value is (see Table 11-1)

$$(1.008665 + 1.007276)\ u = Q + 2.013553\ u.$$

Thus,

$$\begin{aligned}
Q &= 0.002388\ u \\
&= (0.002388\ u)(931.5\ \text{MeV}) \\
&= \underline{2.224 MeV}
\end{aligned}$$

Almost all of this energy is carried away by the photon that is produced in this reaction

Problem 12-19: From Example 12-12, the decay rate from a living organism that contains 175 g of carbon would be

$$(15.6\ \text{decays/g/min})(175\ \text{g}) = 2,730\ \text{decays/min}.$$

The measured decay rate is

$$(8.1\ \text{decays/s})(60\ \text{s/min}) = 486\ \text{decays/min}.$$

In order to find the age of the bone, we need to calculate how many half-lives have passed and then deduce the age. From Example 12-13

$$\left(\frac{1}{2}\right)^n = \frac{486\ \text{decays/min}}{2,730\ \text{decays/min}}$$

or

$$\begin{aligned}
2^n &= \frac{2,730}{486} \\
n\ \ln 2 &= \ln\left(\frac{2,730}{486}\right) \\
n &= 2.49\ \text{half lives}
\end{aligned}$$

Thus, the ago of the bone is

$$\begin{aligned}
t &= n\ t_{1/2} \\
&= (2.49\ \text{half lives})(5730\ \text{y/half life}) \\
&= \underline{14,300\ \text{y}}.
\end{aligned}$$