Physics 390 "Final" Exam Practice Questions

These are a few problems comparable to those you will see on the exam. They were picked from previous exams. I will provide a sheet with useful constants and equations for the exam. The last two pages are the Formula sheets for the Final Exam.

1. (8 pts) The ^{209}Bi nucleus has a spin of 9/2.

(a) (4 pts) What is the angular momentum of this nucleus (in units of $\hbar$)?

(b) (4 tps) Suppose that it were possible to make a beam of completely ionized ^{209}Bi nuclei and pass it through a Stern-Gerlach apparatus. How many distinct "blobs" would you observe on the detector screen?

2. (8 pts) In a scattering experiment it was found that ^{12}C has a nuclear radius of 2.7 fm. The experiment is then repeated with another, unknown element and it is found the the nuclear radius is twice as big. What is the mass number of this unknown element?

3. (8 pts) Consider the following energetically possible transitions of an electron in an atom:

(1) $4p \rightarrow 3p$ (2) $3d \rightarrow 2s$ (3) $4s \rightarrow 2p$ (4) $4d \rightarrow 3p$

Which of these transitions is/are allowed? Explain your reasoning.

4. (12 pts) The proton has a magnetic moment of $\mu_p = 8.8 \times 10^{-8}$ eV/T.

(a) (5 pts) What is the energy difference between the spin-up and spin-down orientations when a proton is placed in a magnetic field of 1.5 T?

(b) (7 tps) In magnetic resonance imaging, a person with a body temperature of 310 K is placed in a 1.5 T magnetic field. Compute the ratio of $N^{\uparrow}/N^{\downarrow}$, where $N^{\uparrow}$ is the number of protons in the spin-up state (parallel to the field) state, and $N^{\downarrow}$ is the number in the spin-down state. State clearly any assumptions you make.

5. (8 pts) If the energy levels in an atom are filled in the sequence 1s, 2s, 2p, 3s, 3p, 4s, 3d.

(a) (5 pts) What is the atomic number of the element that is in its ground state, has all these levels filled, and all higher levels empty?

(b) (3 tps) Suppose that an electron is knocked out of the 1s level. From which of these levels could an electron drop down to fill the resulting unoccupied state? State your reasoning.

6. (10 pts) Sketch the $n = 1$, $n = 2$, and $n = 3$ energy levels for a hydrogen atom in a magnetic field. Indicate three possible *transitions* with solid lines. Indicate three forbidden (or at least highly suppressed) transitions with dotted lines, and state why they are forbidden.
(Ignore fine and hyperfine structure.)

7. (16 pts) When the sun runs out of fusion fuel, it will no longer be able to resist the compression of gravity and will collapse to become a white dwarf star. In this state it will have about the current mass of the Sun (2×10^{30} kg) in a sphere with a radius a bit bigger than that of the Earth (10^7 m say).

(a) (6 pts) Estimate the Fermi energy of electrons in this remnant of the Sun.

(b) (5 tps) Get an order of magnitude estimate for the **total** energy of these electrons. Hint: how does the average energy of electrons compare to the Fermi Energy? Is it roughly the same, or very different?

(c) (5 tps) Given that the pressure can be calculated from $P = -dE_{\text{total}}/dV$, estimate the pressure in this object

8. (6 pts) Protons and neutrons can interact by exchanging pions which are mesons with a rest mass of 140 MeV/c^2. What is the range of this interaction?

Useful constants and equations

$e = 1.602 \times 10^{-19}\ C$ $\quad \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ N\,m^2/C^2$ $\quad \frac{e^2}{4\pi\epsilon_0} = 1.44\ eV\,nm$

$c = 3.00 \times 10^8\ m/s$ $\quad h = 6.626 \times 10^{-34}\ J\,s = 4.136 \times 10^{-15}\ eV\,s$ $\quad \hbar = \frac{h}{2\pi}$

$hc = 1240\ eV\,nm$ $\quad \hbar c = 197.3\ MeV\,fm$ $\quad 1\ eV = 1.602 \times 10^{-19}\ J$ $\quad R_\infty = 1.097 \times 10^7\ m^{-1}$

$m_e = 9.11 \times 10^{-31}\ kg = 511\ keV/c^2 = 5.486 \times 10^{-4}\ u$ $\quad$ neutral ${}^{12}_{6}$C atom mass = 12.0000 u

$m_p = 1.673 \times 10^{-27}\ kg = 938.3\ MeV/c^2 = 1.0073\ u$ $\quad 1\ u = 931.5\ MeV$

$m_n = 1.675 \times 10^{-27}\ kg = 939.6\ MeV/c^2 = 1.0087\ u$

$a_0 = 0.0529\ nm$ $\quad E_0 = -13.6\ eV$ $\quad \alpha = 1/137$ $\quad \sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2K^4}$

Heisenberg: $\quad \Delta p_x\, \Delta x \sim \hbar \qquad \Delta E\, \Delta t \sim \hbar$

Atomic Physics:

Hydrogen Atom: $\quad E = \dfrac{-13.6\ eV}{n^2}$ with $n = 1, 2, 3, \cdots \quad l < n \qquad -l \le m \le +l$

Generalized Balmer Formula: $\quad \dfrac{1}{\lambda} = R_\infty \left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2} \right)$

Angular Momentum: $\quad L^2 = l(l+1)\,\hbar^2 \qquad l = 0, 1, 2, 3, ..$ (orbital)

$L_z = \left|\vec{L}\right| \cos\theta = m\,\hbar \qquad -l \ \le\ m_l \ \le\ l$ (integer steps)

Magnetic Moment: $\quad$ orbital: $\vec{\mu} = \dfrac{q}{2\,M} \vec{L}$

for electron: $\quad |\mu_z| = 2 \cdot |m_s|\ \dfrac{e\hbar}{2\,m_e} = \mu_B = 5.8 \times 10^{-5}\ eV/T$

for proton: $\quad |\mu_z| = g_p \cdot |m_s|\ \dfrac{e\hbar}{2\,m_p} = 5.6\ \dfrac{1}{2}\ \mu_N = 8.8 \times 10^{-8}\ eV/T$

Energy of magnetic dipole in B field: $\quad E = -\mu \cdot B$

Energy of particle in 3-dim ∞ square well: $\quad E_{n_x n_y n_z} = \dfrac{(n_x^2 + n_y^2 + n_z^2) h^2}{8mL^2}$

Molecular excitations: $\quad$ vibrational: $E = (n + \dfrac{1}{2})\,\hbar\omega \qquad$ rotational: $E = \dfrac{L^2}{2\,I} = \dfrac{l(l+1)}{2\,I}\,\hbar^2$

Statistical Physics:

Maxwell Boltzmannc distribution: $f_{MB} = A\ e^{-E/kT}$

Bose-Einstein distribution: $f_{BE} = \frac{1}{B\ e^{E/kT} - 1}$

Fermi - Dirac distribution: $f_{FD} = \frac{1}{e^{(E-E_F)/kT} + 1}$

Boltzmann constant $k = 1.381 \times 10^{-23}\ J/K = 8.617 \times 10^{-5}\ eV/K$

For "gas" of free fermions: $g(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\sqrt{E}$

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3} \qquad E_m = \frac{3}{5}\ E_F$$

Nuclear Physics:

nuclear radius: $R = (1.2 \times 10^{-15}\, m)\ A^{1/3}$ average binding energy / nucleon $\approx 8\ MeV$

range of interaction: $R = \frac{\hbar}{mc} = \frac{\hbar c}{mc^2}$

Decay law: $N = N_0\ e^{-\frac{t}{\tau}}$ with $< t >= \tau = \frac{1}{\lambda}$ and $T_{1/2} = \frac{\ln\ 2}{\lambda}$

Binding energy $\mathrm{B(Z,A)} = [\mathrm{Z}\ \mathrm{m}_p + \mathrm{N}\ \mathrm{m}_n - \mathrm{M}_{atom}(\mathrm{Z,A})]\ \mathrm{c}^2$

Particle Physics:

Baryon: $Q\ Q\ Q$ Meson: $Q\ \overline{Q}$

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}\ \begin{matrix} q = +\frac{2}{3} \\ q = -\frac{1}{3} \end{matrix}$ Leptons: $\begin{pmatrix} e \\ \nu_e \end{pmatrix}\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}\ \begin{matrix} q = -1 \\ q = 0 \end{matrix}$

Cosmology:

Luminosity of star: $L = 4\pi r^2 f$ with $f =$ apparent brightness of star

difference in apparent magnitude: $m_1 - m_2 = 2.5\log(f_1/f_2)$