Some relationships from vector analysis

$$\nabla = \left(\hat{\imath}\ \frac{\partial}{\partial x} + \hat{\jmath}\ \frac{\partial}{\partial y} + \hat{k}\ \frac{\partial}{\partial z}\right):$$ nabla operator

$\nabla \cdot \phi$	$=$ grad ϕ:	ϕ scalar field	$\rightarrow$	vector field
	$= \left(\hat{\imath}\ \frac{\partial}{\partial x}\phi + \hat{\jmath}\ \frac{\partial}{\partial y}\phi + \hat{k}\ \frac{\partial}{\partial z}\phi\right)$			
$\nabla \cdot \vec{A}$	$=$ div $\vec{A}$:	$\vec{A}$ vector field	$\rightarrow$	scalar field
	$= \left(\frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z\right)$			
$\nabla \times \vec{A}$	$=$ curl $\vec{A}$:	$\vec{A}$ vector field	$\rightarrow$	vector field

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right)\hat{\imath} + \left(\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right)\hat{\jmath} + \left(\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right)\hat{k}$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right):$$ laplacian operator

applied on electric potential U:

$\nabla^2\ U =$ div grad $U = 0$ "Laplace equation"

$$\nabla^2\ U = \left(\frac{\partial^2}{\partial x^2}\ U + \frac{\partial^2}{\partial y^2}\ U + \frac{\partial^2}{\partial z^2}\ U\right) = 0$$

Next: $\nabla \times (\nabla \times \vec{A}) = \nabla\,(\nabla \cdot \vec{A}) - \nabla^2\,\vec{A}$

where: $\nabla \cdot \vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z \equiv S$

so that $\nabla\,(S) = \hat{\imath}\ \frac{\partial}{\partial x}S + \hat{\jmath}\ \frac{\partial}{\partial y}S + \hat{k}\ \frac{\partial}{\partial z}S$

and

$$\begin{aligned} \nabla^2\ \vec{A} &= \left(\frac{\partial^2}{\partial x^2}A_x + \frac{\partial^2}{\partial y^2}A_x + \frac{\partial^2}{\partial z^2}A_x\right)\hat{\imath} \\ &+ \left(\frac{\partial^2}{\partial x^2}A_y + \frac{\partial^2}{\partial y^2}A_y + \frac{\partial^2}{\partial z^2}A_y\right)\hat{\jmath} \\ &+ \left(\frac{\partial^2}{\partial x^2}A_z + \frac{\partial^2}{\partial y^2}A_z + \frac{\partial^2}{\partial z^2}A_z\right)\hat{k} \end{aligned}$$