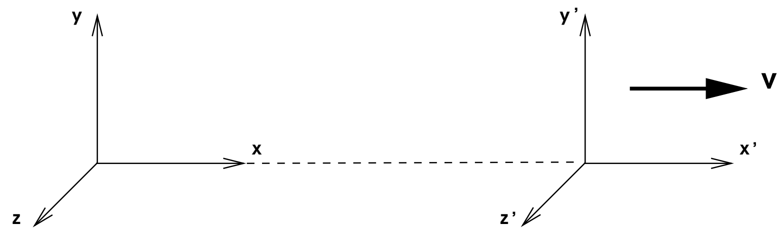
Special Relativity Review



Galilean transformation

Coordinate transformations:

$$
\begin{aligned}
x' &= (x - vt) & x &= (x' + vt) \\
y' &= y & y &= y' \\
z' &= z & z &= z' \\
t' &= t & t &= t'
\end{aligned}
$$

Not consistent with the invariance of the speed of light

Velocity transformations:

$$
\begin{aligned}
{u_x}' &= u_x - v \\
{u_y}' &= u_y \\
{u_z}' &= u_z
\end{aligned}
$$

Energy, momentum:

$\vec{p} = m\vec{v}$
$E = \frac{1}{2}\, mv^2$

Energy, momentum transformations:

Lorentz transformation

$$
\begin{aligned}
x' &= \gamma\,(x - vt) & x &= \gamma\,(x' + vt') \\
y' &= y & y &= y' \\
z' &= z & z &= z' \\
t' &= \gamma\,\left(t - \frac{v}{c^2}x\right) & t &= \gamma\,\left(t' + \frac{v}{c^2}x'\right)
\end{aligned}
$$

when $v/c \ll 1$ Lorentz transformation reduces to Galilean transformation

with $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and $c = 3 \times 10^8$ m/s

$$
\begin{aligned}
{u_x}' &= \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \\
{u_y}' &= \frac{u_y}{\gamma\,(1 - \frac{vu_x}{c^2})} \\
{u_z}' &= \frac{u_z}{\gamma\,(1 - \frac{vu_x}{c^2})}
\end{aligned}
$$

$\vec{p} = \gamma\, m\vec{v}$
$E = \gamma\, mc^2$

$\vec{v},\ \gamma$ refer to particle

$$
\begin{aligned}
{p_x}' &= \gamma\,(p_x - \frac{v}{c^2}E) \\
{p_y}' &= p_y \\
{p_z}' &= p_z \\
E' &= \gamma\,(E - vp_x)
\end{aligned}
$$

where $v,\ \gamma$ refer to motion of primed coordinate system

more Review of Special Relativity

Two important Lorentz invariants (Lorentz scalars):

$$(\Delta s)^2 = (c\Delta\tau)^2 = \underbrace{(c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2}_{\text{if } < 0 \text{ then acausal}}$$

$$(mc^2)^2 = E^2 - (cp_x)^2 - (cp_y)^2 - (cp_z)^2$$

Moving objects appear <u>contracted</u>

$$L = \frac{L'}{\gamma}$$

Moving clocks appear to run more <u>slowly</u>

$$T = \gamma\, T'$$

<u>Doppler shift:</u>

$$\lambda = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\,\lambda' \qquad \text{if source moves away with speed } v$$

$$\lambda = \frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}}\,\lambda' \qquad \text{if source moves towards observer with speed } v$$

<u>General relativity:</u>

Clock at height h above floor appears to run <u>faster</u> by a factor $1+\frac{gh}{c^2}$ where g = gravitational acceleration.