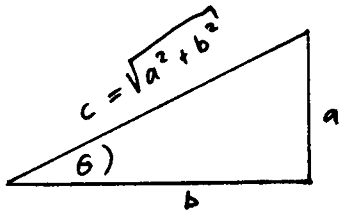
SOME USEFUL MATHEMATICAL RELATIONSHIPS FOR PHYS 390

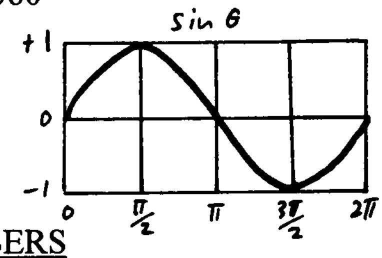
1. TRIGONOMETRY

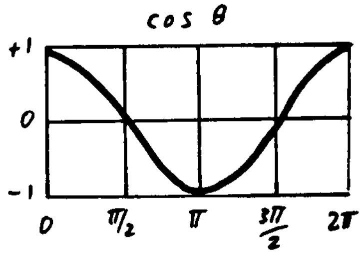
$\sin(A+B) = \sin A \cos B + \sin B \cos A$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$



$\sin^2(\Theta) + \cos^2(\Theta) = 1$
generally measure angles in radians:
2π radian $= 360^\circ$

$\sin\Theta = a / c$
$\cos\Theta = b / c$
$\tan\Theta = a / b = \sin\Theta / \cos\Theta$





2. COMPLEX NUMBERS

$i \equiv \sqrt{-1}$

$z = x + iy$

COMPLEX CONJUGATE $z^* = x - iy$

$|z|^2 = z z^* = x^2 + y^2$ pos. real

also $z = r e^{i\theta} = r(\cos\Theta + i \sin\Theta)$
$z^* = r e^{-i\theta} = r(\cos\Theta - i \sin\Theta)$

so $r^2 = x^2 + y^2$
$x = r\cos\Theta$
$y = r\sin\Theta$

3. CALCULUS

$\frac{d}{dx}(x^n) = n x^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(e^x) = e^x$

$\frac{df(y)}{dx} = \frac{df}{dy}\frac{dy}{dx}$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int (\sin x)\, dx = -\cos x$

$\int (\cos x)\, dx = \sin x$

$\int e^x dx = e^x$

4. SCALAR

is a single number whose value is independent of coordinate system
Examples: temperature, or electric charge at some point in space

5. VECTOR

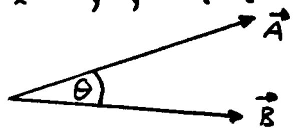
is a quantity whose direction and magnitude, but not its individual components, are independent of coordinate system

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad \text{where} \quad \begin{array}{l}\hat{\imath} = \text{unit vector along x}\\ \hat{\jmath} = \text{unit vector along y}\\ \hat{k} = \text{unit vector along z}\end{array}$$

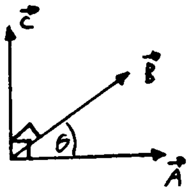
magnitude $A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

a) scalar product of two vectors: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

also $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos \Theta_{AB}$



b) vector product



right-hand rule:
$\vec{C}$ perpendicular to $\vec{A}$, $\vec{B}$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

note $|\vec{A} \times \vec{B}| = |\vec{A}|\;|\vec{B}| \sin \Theta$

6. TAYLOR SERIES EXPANSION of f(x) around x_o

$$f(x) = f(x_o) + \left(\frac{df}{dx}\right)_{at\,x_o}(x - x_o) + \frac{1}{2}\left(\frac{d^2f}{dx^2}\right)_{at\,x_o}(x - x_o)^2 + \cdots + \frac{1}{n!}\left(\frac{d^nf}{dx^n}\right)_{at\,x_o}(x - x_o)^n + \cdots$$

use this to derive approximations when $\Theta \ll 1$ or $x \ll 1$:

$\sin \Theta \approx \Theta$ (with Θ in radian)

$\cos \Theta \approx 1 - \Theta^2 / 2$

$(1 + x)^n \approx 1 + nx$ Ex: $\frac{1}{1+x} \approx 1 - x$

$\sqrt{1+x} \approx 1 + x/2$