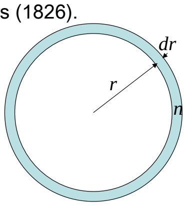
Physics 390
Winter 2007

Introduction to Cosmology

The “Big” Questions

- What is the universe made of?
- What is the history of the universe?
  - Was there a beginning?
  - How and when did it begin?
  - What are the laws that govern its evolution?
- Can we explain the evolution of galaxies, stars, planets and life?
- What will happen to it in the future? Is there an end to time?

Olbers’ Paradox



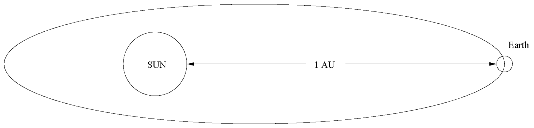
- Why is the night sky dark? Heinrich Olbers (1826). In a static infinite universe the night sky ought to be bright:

$$J = \int_{r=0}^{\infty} \frac{L}{4\pi r^2} 4\pi n r^2 dr = nL \int_{r=0}^{\infty} dr = \infty$$

- Assumptions:
  - unobstructed view (other stars, dust)
  - nL constant (at least no faster than $nL \propto 1/r$)
  - universe is infinitely large (if no stars beyond r_{max}, $J = nLr_{max}$)
  - Universe is infinitely old (for finite age t_0, or no stars older than t_0)
  - Light intensity falls as $1/r^2$

$J \sim nLct_0$

Local Structure



$1AU = 1.5 \times 10^{15}$m

$R_{sun} = 6.96 \times 10^8$m

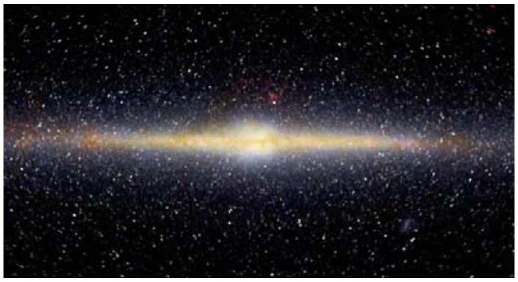
$M_{sun} = 1.99 \times 10^{30}$kg

$R_{earth} = 6.37 \times 10^6$m

$M_{earth} = 5.98 \times 10^{24}$kg

Milky Way Galaxy

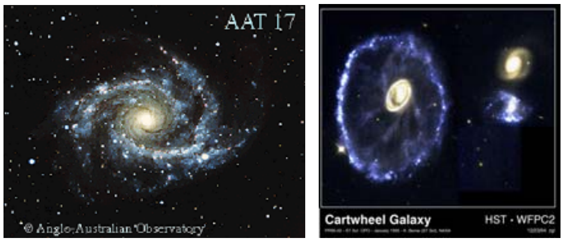
Hubble telescope picture



Milky Way contains ~10^{11} stars in a spiral pancake

Sun located ~2.5×10^{20}m (or 2/3) from center of galaxy

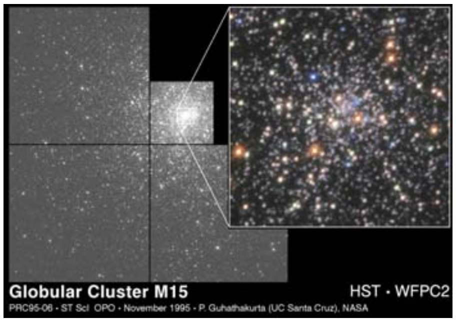
Galaxies



Galaxies come in various sizes and shapes

Tend to be found in clusters

Clusters

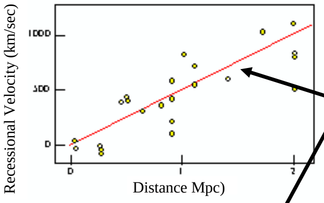


Clusters are concentrated in super clusters

General Relativity

- (1917) Einstein's field equations
  $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \bigcirc = -8\pi G T_{\mu\nu}$$
- Equivalent to Poisson's equation for Newtonian dynamics $\nabla^2 \Phi = 4\pi G \rho$
- GR is a theory of gravity based on the geometry of curved space time. "Matter tells space how to curve and space tells matter how to move."
- (1922) Alexander Friedmann applies Einstein field equations to a homogeneous and isotropic universe (Friedmann Equation).
- Conclusion: A static universe filled with matter cannot exist. It must either be expanding or contracting.
- Einstein puts a cosmological constant, Λ, in his equations to keep the universe from collapsing.

Hubble's Great Discovery - The Universe is Expanding!



$$v = H_o r,$$

$$\frac{\dot{r}}{r} = H_0$$

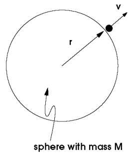
Historical Note: Hubble was off by a factor of 7 in his measurement of H_0 = 500 km/s/Mpc.

Lesson: Beware of systematic errors!

"The expanding universe"

- The expansion of the universe is an expansion of space itself. It is **not** an explosion with pieces flying out from a common center through space.
- The universe is not expanding into anything. It is creating new space between the galaxies as it grows.
- Except for motion due to local gravity through space, each galaxy is at rest with the Hubble flow and "sees" the other galaxies moving away with an apparent speed that increases with distance according to the Hubble law v = H_0r.
- As gravitationally bound structures form, they leave the Hubble flow. Local gravity takes over and drives the continued growth of structure over "billions of years."

Will the universe expand forever?



$$\underbrace{\frac{1}{2}mv^2}_{\text{kinetic energy}} - \underbrace{\frac{GMm}{r}}_{\text{potential energy}} = \underbrace{E}_{\text{total mechanical energy}}$$

If $E > 0$ will expand forever
If $E < 0$ will collapse
The case $E = 0$ is called "critical".

$M = \frac{4}{3}\pi r^3 \rho.$

Substituting the Hubble relation $V = H\,r$ and $M = \frac{4}{3}\pi r^3 \rho_c$, with ρ_c being the critica density, then for $E = 0$ we obtain

$$\frac{1}{2} m\,(H\,r)^2 - \frac{G\,\frac{4}{3}\pi r^3 \rho_c\,m}{r} = 0.$$

or

$$\boxed{\rho_c = \frac{3H^2}{8\pi G},}$$

with $G = 6.67 \times 10^{-11}$ m/(kg s^2) is Newton's constant.

$\rho_c = 10^{-29}$g/cm^3 = 7 H atoms/cm^3 (with H_0=71 km/s/Mpc)

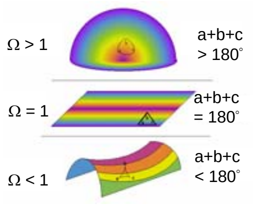
Geometry vs. destiny

- General Relativity: Geometry ⇒ Destiny
- For a universe made entirely of matter the geometry is determined by "Omega" $\Omega = \rho_{tot}/\rho_c$, the ratio between total density of matter and the critical density.

Ω > 1 Positive curvature, closed universe ⇒ eventual collapse.

Ω = 1 Flat space, open infinite universe ⇒ decelerates to rest.

Ω < 1 Negative curvature, open infinite universe, expands forever.



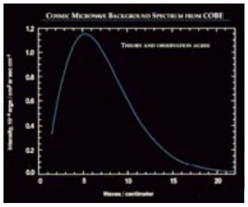
Implications of the Expanding Universe

- The universe had a beginning T ~ r/v = $1/H_0$ (Big Bang).
  T = 13.7 ± 0.2 By (best current value).
- In the past the universe was very hot and dense.
- At sufficiently early times the temperature and density was high enough for all possible particle interactions to occur. Outer Space = Inner Space.
- When T = 6000° atoms formed. The light from this can be seen today as a Cosmic Microwave Background Radiation (CMBR).
- In the first few minutes temperature were hot enough to synthesize light elements (^{4}He, d, ^{3}He, ^{7}Li). Big Bang Nucleosynthesis (BBN). The abundances of these elements in the universe today can be predicted.
- The destiny of the universe will depend upon its content.
- The Hubble expansion, the CMBR and BBN are considered the "three pillars" of Big Bang Cosmology.

The Horizon

- When we look out into the universe we are looking back in time.
- The most distant galaxies are over 10 billion light years away. We are seeing them as they existed 10 billion years ago.
- The "present" exists only locally. We will not "see" the "present" until light has had a chance to reach us.
- The furthest distance that we could see (in principle) is 13.7 billion light years away. This is called our "horizon."
- Our horizon grows at the speed of light.
- The entire observable universe is contained within our horizon. Since nothing can move faster than light, nothing outside our horizon can have had an effect on us at the present time.
- At any given epoch, the largest possible structures are limited by the size of the horizon at that epoch.
- If the universe is decelerating (dominated by matter), larger and larger structures can form as that distance scale enters the horizon.

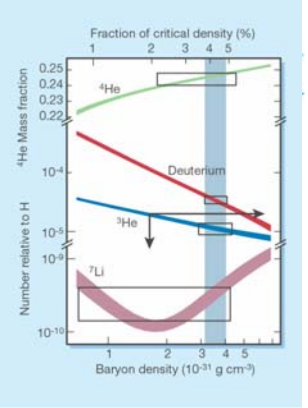
Cosmic Microwave Background Radiation



- As the universe expands it cools.
- Before ~ 300,000 y, radiation dominates the universe.
- Matter density falls as $\rho_m \sim 1/a^3$.
- Radiation density falls as $\rho_{rad} \sim 1/a^4$.
- After ~ 300,000 y, matter dominates over radiation.
- When the temperature falls below 3000K (~380,000 y), nuclei and electrons combine to make atoms and release a burst of light with a characteristic black body spectrum.
- With few ionized atoms, the universe is transparent for the first time.
- The universe has since expanded by a factor of ~1000. The radiation is now peaked in the microwave region with a blackbody temperature of only 2.7K .
- This CMBR is everywhere around us. It was first observed by Arno Penzias and Robert Wilson at Bell Labs.
- The FIRAS instrument on COBE confirmed the black body shape and measured the CMBR temperature at 2.725 ± 0.001K

Big Bang Nucleosynthesis

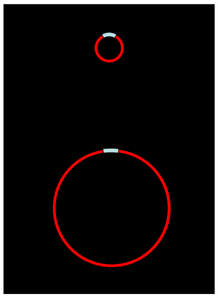
- $n \rightarrow p + e^- + \bar{\nu}_e$ with a half life of 10.3 min.
- In the first few minutes temperatures were hot enough to make light nuclei (^{4}He, d, ^{3}He, ^{7}Li).
- The abundances depend critically on the density of baryons (n + p).
- Measured abundances in the universe agree with predictions if baryons make up ~ 4.4% of ρ_{crit} today.



Problems with the Big Bang

- The Horizon Problem
  - CMBR measurements show that the temperature is the same in all directions.
  - If we follow the Hubble expansion back in time, regions of the sky separated by more than a few degrees could have never been in contact with each other and could not have come into thermal equilibrium.
- The Flatness Problem
  - If $\Omega \neq 1$, it rapidly evolves away from $\Omega = 1$ as the universe expands.
  - If Ω is close to unity now, it had to be at most within 10^{-28} of unity (0.9999999999999999999999999999) near the beginning of time (BBN).
  - Only if Ω is precisely = 1 would it remain = 1.
  - What mechanism could fine-tune the universe to this degree?

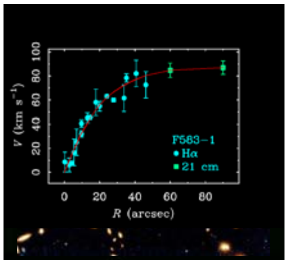
Inflation



- In 1979 Alan Guth proposed an inflationary universe to solve the "horizon problem" and to explain the apparent "flatness" of the universe.
- From ~10^{-35} - 10^{-33} s, the inflationary universe doubled in size every 10^{-35} s, expanding by a factor of 10^{28} and setting $\Omega = 1$.
- Quantum fluctuations in the early universe were expanded into the "seeds" that nucleated the large-scale structures in the universe today.
- Inflation was powered by a time-dependent vacuum energy that transformed itself into all the matter and energy in the universe today.

Adding up the Matter and Coming Up Short

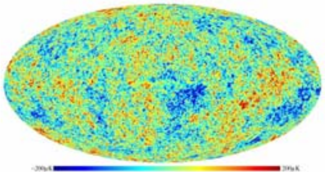
- Counts of luminous objects (stars, galaxies, galaxy clusters) have shown that only ~ 0.5% of ρ_{crit} is made up of bright stars.
- Evidence for the existence of significant amounts of dark matter comes from:
  - Galactic rotation curves
  - Velocity distributions of galaxies in galaxy clusters
  - Measurements and simulations of large scale structure
  - Weak gravitational lensing mass census
- All show that the universe contains significant amounts of dark matter (up to 27% of ρ_{crit}).
- Not nearly enough for $\Omega = 1$.

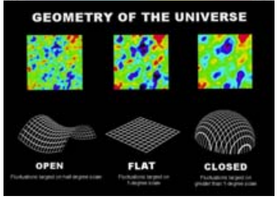


$$\frac{mv^2}{r} = \frac{GM_r m}{r^2} \Rightarrow v = \sqrt{\frac{GM_r}{r}}$$

Fluctuations in the CMBR

- Within the last 1.5 years the Wilkinson Microwave Anisotropy Probe (WMAP) has measured fluctuations in the CMBR on angular scales as fine as 0.1°.
- The hot and cold spots correspond to regions of high and low density at the time of recombination.
- The dominant angular scale is sensitive to the geometry of the universe.
- They conclude that the universe is flat.
  $\Omega_{total} = 1.02 \pm 0.02$





Dark Energy and Acceleration

- We seem to live in a flat universe ($\Omega_{total} = 1.02 \pm 0.02$):

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = E = 0 \quad \text{with} \quad M = \rho\left(\frac{4}{3}\pi r^3\right), \quad \text{with} \quad \rho = \text{ constant}$$

Then

$$v = \frac{dr}{dt} = \sqrt{\frac{8\pi}{3}G\rho}\; r = \frac{1}{\tau}r \qquad \text{or} \qquad \int_{r_0}^{r}\frac{dr}{r} = \int_{0}^{t}\frac{dt}{\tau} = \ln\frac{r}{r_0} = \frac{t}{\tau}$$

leads to $r = r_0 e^{\frac{t}{\tau}}$

or exponential expansion of the universe.