Classical Electromagentism Review

1.) Force acting on a charged particle is

$$\vec{F} = q\,\vec{E} + q\,\vec{v} \times \vec{B},$$

where q = charge, and $\vec{v}$ = velocity of particle.

2.) Maxwell's Equations (differential form)

$$\begin{aligned}
\text{a)}\quad \nabla \cdot \vec{\mathrm{E}} &= \frac{\rho}{\epsilon_0} \quad \text{(Gauss' law)} \\
\text{b)}\quad \nabla \cdot \vec{\mathrm{B}} &= 0 \quad \text{(no monopoles)} \\
\text{c)}\quad \nabla \times \vec{\mathrm{E}} &= -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law of induction)} \\
\text{d)}\quad \nabla \times \vec{\mathrm{B}} &= \mu_0\,\epsilon_0\,\frac{\partial \vec{E}}{\partial t} + \mu_0\,\vec{J} \quad \text{(Ampere's law)}
\end{aligned}$$

where $\rho = \frac{\text{charge}}{volume}$, $\vec{J} = \frac{\text{current}}{perp.\ area}$, $\epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{N\,m^2}$ is the permitivity, and $\mu_0 = 4\,\pi \times 10^{-7} \frac{\mathrm{N}}{A^2}$ the permeability.

For $\rho = 0$, and $\vec{J} = 0$, so Maxwell's Equations become (notice symmetry between $\vec{E}$ & $\vec{B}$):

$$\begin{aligned}
\text{a)}\quad \nabla \cdot \vec{\mathrm{E}} &= 0 \\
\text{b)}\quad \nabla \cdot \vec{\mathrm{B}} &= 0 \\
\text{c)}\quad \nabla \times \vec{\mathrm{E}} &= -\frac{\partial \vec{B}}{\partial t} \\
\text{d)}\quad \nabla \times \vec{\mathrm{B}} &= \mu_0\,\epsilon_0\,\frac{\partial \vec{E}}{\partial t}
\end{aligned}$$

Using an identity from vector calculus, we can derive the wave equations for $\vec{E}$ & $\vec{B}$. Taking e.g. Eq. b) and operate with $\nabla\times$ from the left, leads to:

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\,(\underbrace{\nabla \cdot \vec{E}}_{=0}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\underbrace{\nabla \times \vec{B}}_{\mu_0\,\epsilon_0\,\frac{\partial \vec{E}}{\partial t}}) = -\mu_0\,\epsilon_0\,\frac{\partial^2 \vec{E}}{\partial t^2}.$$

This is the wave equation for $\vec{E}$: $\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\mu_0\,\epsilon_0}\,\nabla^2 \vec{E} = 0}$

The general form is, assuming $f = f(x)$ to simplify the math: $\boxed{\frac{\partial^2 \vec{f}}{\partial t^2} - c^2\,\frac{\partial^2 \vec{f}}{\partial x^2} = 0}$.

This is a second order, partial differential equation which is satisfied by any function f, where $f = f(x,t) = f(u) = f(x \pm ct)$ (you will prove that in your HW).

3.) Propagation of e-m waves in free space:

A fixed point of f is represented by: $x \pm ct = \text{const}$. Then

$$\frac{\mathrm{d}}{\mathrm{dt}}\,(x \pm ct = \text{const}) = \frac{\mathrm{dx}}{\mathrm{dt}} \pm c = 0,$$

or

$$\frac{\mathrm{dx}}{\mathrm{dt}} = \text{ velocity of propagation } = \pm c.$$

This means that e-m waves propagate in free space with the speed of light.

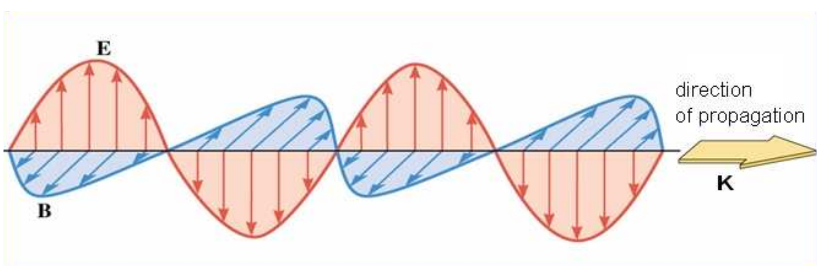
4.) Electromagnetic Plane Waves: (solutions of the wave equation)

$$\vec{E} = \vec{E}_0 \,\cos(K \cdot \vec{r} - \omega t)$$

$$\vec{B} = \vec{B}_0 \,\cos(K \cdot \vec{r} - \omega t)$$

where $\vec{K} \times \vec{E} = \omega \vec{B}$, $K = 2\pi/\lambda$ (with λ = wavelength), $\omega = Kc = 2\pi\nu = 2\,\pi/T$ (with ν = frequency, and T = period).

$$\vec{S} = \frac{1}{\mu_0}\,\vec{E} \times \vec{B} = \frac{\text{energy}}{\text{area}}/\text{time}$$



Properties:

1. $\vec{E} \perp \vec{B}$
2. transverse wave: $\vec{E}$, $\vec{B} \perp \vec{K}$
3. $\vec{E}$ and $\vec{B}$ are in phase
4. waves are polarized
5. $|\vec{B}| = \left|\frac{\vec{E}}{c}\right|$