Physics 390: Homework set #3

Due Friday February 10, 2006

Reading: Tipler & Llewellyn, Chapter 6

Questions:

1. If the ground-state energy of an electron in a box were of the same magnitude as hydrogen in the ground state, how would the width of the box compare to the Bohr radius?

2. (a) In the systems we have considered, do any two energy eigenfunctions that are proportional to each other have the same energy eigenvalues? (b) Can you find two wave functions that are not proportional to each other, yet still have the same energy eigenvalues? What distinguishes the functions?

3. Does Equation 6-27 imply that we know the momentum of the particle exactly? If so, what does that imply about our knowledge of its position? How can you reconcile this with our knowledge that the particle must be in the well?

4. In this problem you will verify the Heisenberg uncertainty relations for the ground state of the quantum harmonic oscillator, for which (Equation 6-58)

$$\psi_0(x) = A_0 E^{-m\omega x^2/2\hbar}.$$

(a) Show that the normalization condition $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$, requires $A_0 = (m\omega/\hbar\pi)^{1/4}$.

(b) What value do you expect for $\langle x \rangle$? Use a symmetry argument rather than a calculation.

(c) Compute $\langle x^2 \rangle$. Then compute $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

(d) What value do you expect for $\langle p \rangle$? Use a symmetry argument rather than a calculation.

(e) Conservation of energy for the harmonic oscillator can be used to relate p^2 to x^2. Use this relation, along with the value of $\langle x^2 \rangle$ from part (c), to find $\langle p^2 \rangle$.

(f) Using the results of parts (d) and (e), evaluate Δp.

(g) Finally, using the results of parts (c) and (e), evaluate $\Delta p \Delta x$ for the harmonic oscillator. Is the result consistent with the uncertainty relationship?

Problems: 3, 6, 25[1], 42, 50

[1] Hint: Assume that $V_1 < E < V_2$.