Some relationships from vector analysis

$$\nabla = \left(\hat{\imath}\, \tfrac{\partial}{\partial x} + \hat{\jmath}\, \tfrac{\partial}{\partial y} + \hat{k}\, \tfrac{\partial}{\partial z} \right) : \qquad\qquad\qquad \text{nabla operator}$$

$$\nabla \cdot \phi \qquad = \text{grad } \phi: \qquad\qquad \phi \ \text{ scalar field} \qquad \rightarrow \qquad \text{vector field}$$
$$= \left(\hat{\imath}\, \tfrac{\partial}{\partial x}\phi + \hat{\jmath}\, \tfrac{\partial}{\partial y}\phi + \hat{k}\, \tfrac{\partial}{\partial z}\phi \right)$$

$$\nabla \cdot \vec{A} \qquad = \text{div }\ \vec{A}: \qquad\qquad \vec{A}\ \text{ vector field} \qquad \rightarrow \qquad \text{scalar field}$$
$$= \left(\tfrac{\partial}{\partial x}A_x + \tfrac{\partial}{\partial y}A_y + \tfrac{\partial}{\partial z}A_z \right)$$

$$\nabla \times \vec{A} \qquad = \text{curl }\ \vec{A}: \qquad\qquad \vec{A}\ \text{ vector field} \qquad \rightarrow \qquad \text{vector field}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \tfrac{\partial}{\partial x} & \tfrac{\partial}{\partial y} & \tfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\tfrac{\partial}{\partial y}A_z - \tfrac{\partial}{\partial z}A_y \right)\hat{\imath} + \left(\tfrac{\partial}{\partial z}A_x - \tfrac{\partial}{\partial x}A_z \right)\hat{\jmath} + \left(\tfrac{\partial}{\partial x}A_y - \tfrac{\partial}{\partial y}A_x \right)\hat{k}$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2} + \tfrac{\partial^2}{\partial z^2} \right) : \qquad\qquad \text{laplacian operator}$$

applied on electric potential U:

$$\nabla^2 U = \text{div grad } U = 0 \qquad\qquad\qquad \text{"Laplace equation"}$$

$$\nabla^2 U = \left(\tfrac{\partial^2}{\partial x^2}U + \tfrac{\partial^2}{\partial y^2}U + \tfrac{\partial^2}{\partial z^2}U \right) = 0$$

Next: $\qquad \nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

where: $\qquad \nabla \cdot \vec{A} = \tfrac{\partial}{\partial x}A_x + \tfrac{\partial}{\partial y}A_y + \tfrac{\partial}{\partial z}A_z \equiv S$

so that $\nabla(S) = \hat{\imath}\, \tfrac{\partial}{\partial x}S + \hat{\jmath}\, \tfrac{\partial}{\partial y}S + \hat{k}\, \tfrac{\partial}{\partial z}S$

and

$$\nabla^2 \vec{A} = \left(\tfrac{\partial^2}{\partial x^2}A_x + \tfrac{\partial^2}{\partial y^2}A_x + \tfrac{\partial^2}{\partial z^2}A_x \right)\hat{\imath}$$
$$+ \left(\tfrac{\partial^2}{\partial x^2}A_y + \tfrac{\partial^2}{\partial y^2}A_y + \tfrac{\partial^2}{\partial z^2}A_y \right)\hat{\jmath}$$
$$+ \left(\tfrac{\partial^2}{\partial x^2}A_z + \tfrac{\partial^2}{\partial y^2}A_z + \tfrac{\partial^2}{\partial z^2}A_z \right)\hat{k}$$