Midterm #4 Practice Exam Questions

Topics in relativity:
- Michelson-Morley and ether drift
- Postulates of special relativity
- Simultaneity
- Lorentz transformations
- Length contraction
- Relativistic velocity addition
- Relativistic Doppler effect
- Spacetime intervals and causality
- Energy momentum invariant
- Relativistic momentum
- Relativistic total energy
- Rest energy
- Massless particles

1: Two events occur in the stationary frame S with coordinates:
   1: \( x = x_0, \ t = x_0/2c \)
   2: \( x = 4x_0, \ t = x_0/c \)
Assume that the origins of S and S' are the same at \( t=t'=0 \). What velocity must a reference frame S' be travelling for these two events to occur at the same time in S'?

2: A pair of atomic clocks are synchronized on the ground. Clock A is then placed in an airplane which flies in circles above clock B at constant velocity \( v=300 \text{m/s} \) for 1 week, stopping only briefly to refuel. Estimate the time difference between the two clocks.
3: Two events are simultaneous, but separated by a distance $x_0$ in one reference frame $S$. What velocity must reference frame $S'$ be moving with to cause the two events to occur in the same place? In other words, what velocity is needed to make $\Delta x' = 0$.

4: People sometimes imagine travelling to nearby stars by accelerating to something near the speed of light gradually and then coasting. Imagine that our spacecraft reaches a speed of 0.95$c$ relative to the galaxy as a whole. As it travels through interstellar space it collides with hydrogen atoms which are at rest relative to the galaxy. The rest mass of hydrogen is about 0.94 GeV. What is the energy of each hydrogen atom in the reference frame of the spacecraft?

5: What fractional error does one make when calculating the kinetic energy using the Newtonian equation $KE=1/2mv^2$ instead of the relativistic equation for an object travelling with a speed of $10^4$m/s?

6: Two events occur at the same time in reference frame $S$ and are separated by 1km along the x axis. What is the time difference between these two events when measured in a reference frame $S'$ moving with constant velocity along x if the spatial separation in $S'$ is 2km?
7: Consider two reference frame S, and S' moving with velocity v along the x axis of S. If a particle travels in reference frame with \( u_x = c \), what is its velocity \( u_x' \) in reference frame S'?

8: A set of \( \pi^0 \) mesons is produced with total energies ranging from \( 6.0 \times 10^9 \) to \( 18.0 \times 10^9 \) eV. The rest energy of the \( \pi^0 \) is \( 135.1 \times 10^6 \) eV and its lifetime measured in its rest frame is \( 2 \times 10^{-16} \) s.

   a) What is the range of speeds of these pions?
   b) What is the range of distances through which they travel (on average) before decaying?

9: Is the classical concept of an incompressible fluid valid in relativity?

10: Take the rest radius of the Earth to be 6400 km, and its orbital speed around the sun to be 30 km/s. By how much is the Earth's radius foreshortened when seen by an observer on the Sun?