Aspirational Bargaining[*]

(under revision)

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Abstract

Assume that two impatient individuals wish to divide a unit pie. This paper offers a noncooperative resolution of this classic complete information bargaining problem *without procedures.* To do away with the reliance on exogenous procedures, we formulate the game in continuous time, with unrestricted timing and content of offers. To deal with the multiple equilibria, we employ a standard refinement in repeated games that admits a simple behavioural interpretation: Reprising experimental work from 1960, we introduce and explore *aspirational equilibrium* — a Markovian refinement of subgame perfection where behaviour is governed by aspiration values (expected payoffs). The analysis is tractable, and generates many intuitive aspects of bargaining.

We find that discounted aspiration values form a martingale, and thereby compute bounds on the expected bargaining duration. We also deduce some simple implications about consecutive offers, and relate delay times, offers, and acceptance rates. Finally, we draw into question a traditional comparative static: *Ceteris paribus,* more impatient players can expect more of the pie.

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1 Introduction

The classic bargaining problem presents two individuals with the opportunity to share a dollar if they can first agree on a partition. The rebirth of interest in this subject from a modern noncooperative approach dates to Rubinstein (1982), itself related to Ståhl (1972). He proposes a discrete time alternating offer model with payoff discounting.

We shall focus on three critical "realistic" elements of the above bargaining problem: (*i*) two risk neutral players $1, 2$ must divide a unit pie, by concurring on a feasible split; (*ii*) an agreement means a proposal by one player and an immediate acceptance by the other; (*iii*) the force for timely resolution is the players' impatience. We shall ignore the additional (and important) element of incomplete information.

This paper explores the bargaining problem without temporal monopoly, and instead aims for a behaviourally motivated noncooperative solution. Since discrete time forces the unique Rubinstein subgame perfect equilibrium (SPE), we shift to continuous time. View a discrete time model, as a restriction of the feasible action space to a subset of continuous time. We instead build the behavioural foundation into the solution concept. This Markovian refinement of SPE — *aspirational equilibrium* (AE) — is based on a single unifying idea: *Players' behaviour at any moment is governed by their current expected payoffs — and* only *their expected payoffs.* Motivated by earlier empirical work in behavioral bargaining, we call these expected payoffs *aspiration values*. We shall argue that:

- The above unifying idea captures many intuitive properties of bargaining;
- The formal model employs well-studied Markovian strategies from repeated games, strengthened by assuming time stationarity (required by the freedom of continuous time);
- The refinement has strong behavioral foundations for bargaining;
- We derive many new falsifiable results about bargaining — true in all equilibria;
- The theory is fundamentally different from existing bargaining work.

A. The Intuitive Structure of Aspirational Equilibria. For a flavour of this refinement, consider an AE without immediate agreement. As strategies depend on expected payoffs alone (and so not on time), delay owes solely to the players' randomization. With strict time preference, there are rents from ending this bargaining hiatus. And since any player is indifferent about proposing, only the opponent strictly benefits from this offer. In other words, any delay implies that the players are locked in a *war of attrition*: Each strictly prefers that his counterpart and not himself stop the clock and propose. So an endogenous *'proposee' advantage* arises, as receiving an offer makes one better off.

Next consider what transpires when some player, say Mr. 1, finally tenders an offer to Mr. 2. The latter might accept it with probability one. If so, game over. Assume not. Agreement brings us to the Pareto frontier, and is efficient, while rejection incurs further delay, and is inefficient. Since Mr. 2 weakly prefers to reject, Mr. 1 must be disappointed by this outcome, suffering a strict payoff loss if Mr. 2 declines, and payoff jump otherwise. This yields several desirable and realistic bargaining features: (*i*) wars of attrition explain negotiation lags; (*ii*) serious offers are concessions; (*iii*) offers may be turned down; (*iv*) proposers are strictly disappointed from rejection, and strictly pleased by acceptance. These intuitive features are not a priori assumed, but fall out of the refinement. Critically, these features are true of all of the equilibria.

This connection with the war of attrition is quite natural. Indeed, that classic timing game shares two critical features of the bargaining model. First, for a given final outcome (i.e. the pie split or player who stops), players agree that the passage of time is costly. So in the time dimension, the players are engaged in a pure coordination game. Second, fixing the elapse time of the game, the players are engaged in a perfectly competitive game. On the other hand, the outcome of the war of attrition is merely a 0-1 variable (i.e. who stops), while a continuous pie split in $[0, 1]$ emerges from the bargaining problem. What we do can thus be viewed as taking the classic war of attrition, and first endogenizing the stopping payoff with a bargaining proposal; second, we allow this payoff to be rejected, and thus another war of attrition to recommence, etc. until agreement. In this sense, our model of bargaining is two steps removed from the classic war of attrition. Any richness of the model owes to the interplay of these additional two features.

B. Contrast with Temporal Monopoly. Harsanyi (1956) observed: "As is well-known, ordinary economic theory is unable to predict the terms on which agreements tend to be reached in cases of ... bilateral monopoly. Only on the basis of additional *assumptions* does the theory of games furnish a determinate solution." Rubinstein's assumptions about the action space converted the intractable bilateral monopoly into a simple sequence of *temporal monopolies:* Players are *exogenously* given the power to ask the other party to accept an offer, or to burn some of the pie by declining. This yielded a unique SPE with immediate agreement favouring the proposer. This hard-wired proposer advantage is precisely opposite to our *endogenous proposee advantage*, and is the ultimate source of the different implications between these approaches to bargaining.

Temporal monopoly in no way precludes endogenous timing — as Perry and Reny (1993), Sakovics (1993), and Stahl (1993) have cleverly demonstrated. They posit continuous time bargaining games where players face tiny 'waiting' and 'reaction' times after offers. This temporal monopoly setting yields very small monopoly rents, and thereby an accordingly small proposer advantage. The intuitive predictions of our model thus do not obtain — wars of attrition and strict concessions. Since their results intuitively obtain for random but boundedly positive *mean* waiting times after offers, at first blush one might presume that AE are merely special cases. But our timing is truly unrestricted — and to the player with an aspiration level approaching its highest level, offers *must* be tendered arbitrarily quickly (Theorem 3).

Temporal monopoly captures situations where the time cost of each negotiation round plays a central role in the players' minds. While there are many situations with this key feature, this effect intuitively should play no role in many settings. Players may well care about delay, and yet consider the reply time unimportant for this delay. Rather, from their viewpoint, delay may owe solely to long elapse times without any offers made.

One could argue that nonetheless some reply delay lag time always *does exist*. To this correct observation, we appeal to the sage modelling advice in Rubinstein (1991), attributed to Aumann: "The most basic principle in the art of formal modelling" is that the constructed game capture the players' understanding of the game. The illustrative cited example is that a finitely repeated game may be an inappropriate model of finite-lived players "unless the finite period enters explicitly into the players' considerations." In other words, one must balance the positive descriptive aspects and any underlying behavioural elements. So irrespective of the reality of small time lags, temporal monopoly need not be the right model for all contexts. Indeed, the *slightest amount* of temporal monopoly switches our endogenous proposee advantage to the proposer advantage, thereby reversing most results. We next argue that replacing procedures by our psychologically-based refinement is an important road to explore.

C. Falsifiable Implications of our Theory. Many bargaining papers, even with complete information, have provided stories justifying that delay can occur. We take this as a given, and it arises naturally here. Rather, our focus is on the *bargaining process*, where we are able to link the offer timing, content, and acceptance rates. For instance, we deduce the expected bargaining duration from offers. Theories must make clear predictions. Typically, this means a unique equilibrium, but in our case, this is neither possible, nor desirable (as we argue). Instead, we are able to make *unique predictions* about many aspects of the bargaining process *true across all equilibria.*

First, complete information bargaining papers are often focused around a one-shot prediction of the bargaining outcome. Before seeing any offer, we are unable to offer such a prediction; after seeing alternating offers, we can. Indeed, an AE specifies exogenously-given aspiration levels for the players — the initial state of a Markov process. Once Mr. 1 offers x, Mr. 2's aspiration value jumps to this new level x. Afterwards, the concession that is offered may yet be rejected. (See Figure 1.) This leads to a tractable Markov and martingale stochastic process on the space of possible pairs of discounted aspiration levels (Theorem 4). We can then conclude that bargaining almost surely ends in finite time; we also use this process to provide a simple lower bound on the duration of bargaining based upon observed alternating offers (Corollaries 3–4).

We explore some key links between when and what to offer, and what to accept. The second of consecutive offers by a player is more generous (Corollary 5). Also, incentive constraints force a trade-off between the offer content and delay time (Corollary 6), as well as the chance an offer is rejected and the surplus it concedes (Corollary 7). We show how some properties — sweetening an offer raises its acceptance chance (Corollary 8), for one — turn on a decreasing or convex aspiration set, which we also characterize (Lemma 4–5).

The distinction between a proposer and proposee advantage yields an enlightening contrast between the models. In our final result, we explore what happens if player 1 becomes more impatient. In the temporal monopoly framework, this increases player 2's temporal advantage, and forces both players to propose pie splits more favourable to player 2. In our aspirational framework, notwithstanding the multitude of equilibria, we believe that there is insight gained from posing this question. At the very least, it questions the validity of the conclusion in the temporal monopoly world. In a word, we ask what happens to the whole equilibrium set. We argue that there is a natural bijection between AE in the two worlds where player 1 is more and less impatient. Provided player 2 offers more rapidly, all incentive conditions are restored at the current aspiration levels. With this mapping, we can see that the final outcome splits in the AE with a more impatient player 1 place player 2 more often in the strategically disadvantageous offering role. We show in Theorem 5 that this raises the expected ultimate pie share of player 1.

D. Contrast with Bargaining Under Incomplete Information. We first acknowledge a key similarity. Our model offers a useful continuity with the literature on bargaining under incomplete information, where the norm is that players shy away from proposing, as they engage in a war of attrition. We are thus able to understand the emergence of such wars in settings where incomplete information seems a stretch.

But our proposee advantage owes not to the fear of information revelation, but because of the simple intuitive fact that offers become behavioral benchmarks for future play. There is no fear of revealing any information. This is consistent with many union-management bargaining episodes, where the concern is not so much over information revelation, but rather for the psychological ramifications of appearing weak.

Our model is *globally time-stationary* — that is, the states are all communicating; the game might even return to its initial state. Indeed, ours is not a story of a *single* war of attrition, but an endogenous sequence. Our AE generally have multiple offer episodes not by the same player. Each proposal shifts the balance of power towards the other player, should bargaining continue. Since mixed strategies govern the delay, multiple reversals of fortune can occur. Incomplete information bargaining models by contrast typically have one or both parties engaged in an incomplete information war of attrition: their offer reveals either their type, or their type range. The model can never return to its pristine state. Information revelation, by its very nature, is an inherently irreversible process.

From a theoretical perspective, a globally time-stationary model is extremely *simple* to analyze by comparison. The brevity of our main characterization result and our examples speak for themselves. The role of mixed strategies and not uncertain types also makes better sense of a subject of historic interest: bargaining power. Harsanyi (1956) opined that random elements may be the source of bargaining strength: "Of course, information on the two parties' . . . strength alone may not suffice: the outcome may depend significantly on such 'accidental' factors as . . . bargaining skill." This view is also well-captured by our model: Having randomly offered and been rejected first, perhaps many times, hurts one's bargaining position.

Our final comparative static (Theorem 5) is also unlike any in the literature on bargaining under incomplete information. Conceptually, this result is closer to the theory of the war of attrition, which we have used. But players only entertain a single action the war of attrition, namely whether to stop. Our offer choice renders this a far richer game.

E. Theoretical Support for the Aspirational Refinement. Our Markovian refinement is based on established work in repeated games by Abreu, Pearce, and Stacchetti (1986). APS (1986) characterized subgame perfect and sequential equilibrium payoff sets, and in so doing, employed a Markovian structure that replaced each subgame by its associated expected payoff. The refinement obviously did not refine the equilibrium payoff set, while APS did not focus on the behavioral implications. Nonetheless, this refinement has been adopted by the imperfect monitoring repeated games literature. Our notion of an AE must also deal with continuous time — a feature not present in the repeated games literature. The problem is that even if strategies are functions of expected payoffs, these aspirations may move between offers with the passage of time. For instance, time delay itself may be a negative signal, discouraging the players. Essentially, even though strategies only depend on expected payoffs, the latter are functions of time, and so time enters by the backdoor as a key consideration. This complication is worthy of study, and possibly relevant, but in this first exploration of AE, we simply impose *time stationary* strategies. This restriction is consistent with our single unifying idea that expected payoffs are the *only* consideration we shall focus on.

Continuous time models introduce many well-known problems, most especially the twin issues that (*i*) outcome profiles need not be well-defined given strategies, and (*ii*) best replies need not exist. A key advantage of our assumption of time-stationary strategies that we prove that it simultaneously resolves both of these technical issues.

F. Experimental Support for the Refinement. That expectations matter in bargaining should be intuitively compelling. It may come as no surprise then that it agrees with long-established work at the nexus of economics and psychology.

The salience of aspirations in decision-making has long been established in the psychology literature (Siegel 1957). In fact, its significance in bargaining has long ago been investigated in laboratory tests. Siegel and Fouraker (1960) studied a simple buyer-seller bargaining game. The project's goal was to investigate the role of differential information on the bargaining split, seeing whether the better-informed party excelled. But among the "interesting implications", they conclude that "the basis of both the bargainer's 'expectancy' and, at least partially, of his 'bargaining strength' may very well be his *level of aspiration*" (p. 60). While the authors expected the 50-50 equal split under complete information, their experiments showed how aspirations acted as a dynamic anchor on the bids made, and explored how the exchange of offers affected these aspirations. A key role played by offers in an AE — to ratchet up the opponent's aspiration level — was observed in a specific experiment (pp. 80–81), as was the intertemporal non-monotonicity of a player's offer (pp. 77–90). Roth and Schoumaker (1983) have since underscored the importance of expectations in bargaining.

Relative to the literature, we take some liberty in our use of the term "aspiration." We take Siegel and Fouraker's phrase 'expectancy' literally, as our aspiration is a rational expectation and not a purely arbitrary benchmark or "reference point" against which gains and losses are compared (Gilboa and Schmeidler (1994)). For we wish to provide a non ad hoc basis for the aspirations discussed by the psychological studies. This gives the desired dynamic anchor for behavior sought by Siegel and Fouraker. A few papers (recently, Karandikar, Mookherjee, and Ray (1998)) have studied models where aspirations evolve over time. With our rational aspirations, the evolution is endogenous to the model.

The growing importance of the behavioral economics literature is well-known. Here, we use the behavioural economic approach to restrict the set of equilibria determined by rationality considerations alone, rather than to overturn the predictions of rationality. This use of the behavioral approach appears to be new.

Outline. In section 3, we develop our aspirational refinement of subgame perfection. Some immediate properties are explored in section 4, and sensitivity analysis is performed in section 5. The conclusion in section 6 links some of the details of our approach to the literature. We appendicize any technical proofs.

2 The Continuous Time Bargaining Model

A. Overview. Two players $i \in \{1, 2\}$ (often denoted i, j) must split a unit pie. The players are impatient, discounting future payoffs by possibly different positive interest rates $r_1, r_2 > 0$. The players' outside options are each zero. All parameters are common knowledge. Bargaining transpires in real time on $[0, \infty)$. At a time of his choosing, either party may propose a pie split. Proposals must lie on the *Pareto frontier* where pie shares sum to one — and so may be summarized by the share $x \in [0, 1]$ offered to the other party. To capture the irreversibility of tendering an offer and the risk of rejecting, offers are assumed final and 'exploding': They must be immediately either accepted — thereby ending the game — or rejected, with no explicit future commitment. (On the other hand, we shall see that in equilibrium, rejected offers will have *implied* future repercussions.) There are some delicacies with the handling of simultaneous offers, and the existence of well-defined strategy profiles. These are quite carefully dealt with in the appendix, but can safely ignored for now, since they only affect zero chance events.

B. Payoffs and Subgame Perfection. The payoff to a pure strategy profile σ with $h^\sigma \in \mathcal{H}_Y$ is the vector $\pi(\sigma) = (\pi_1(\sigma), \pi_2(\sigma))$, where $\pi_i(\sigma) = e^{-r_i t}(1-x)$ and $\pi_j(\sigma) = e^{-r_j t}x$ if player i made the final offer x at time $t = T_1(h^\sigma) < \infty$ and player j accepted it. Otherwise, if bargaining lasts forever, or no proposal occurs after some rejection, then $\pi(\sigma) = 0$. The payoff $\pi(\sigma)$ of a behavior strategy σ is defined by taking expectations.

A behavior strategy profile σ is a *subgame perfect equilibrium* (SPE) if for all $h \in \mathcal{H} \setminus (\mathcal{H}_i \cup \mathcal{H}_Y)$, any $(s,t) \in \text{supp}(\sigma_i(h))$ is a best reply to σ_j at h. This paper is in fact entirely focused on a Markovian class of subgame perfect equilibria. To step back from the abstraction and fix ideas, we now give a simple example that we later revisit.

Example (Constant Acceptance Rate): In the example SPE, whenever one player's *aspiration value* (his expected payoff) is v_i, the other's will be $\varphi(v_i)$, where $\varphi : (0,1) \to (0,1)$ is the strictly decreasing function $\varphi(v_i) = (1-v_i)/(1+v_i)$. Note that $\varphi = \varphi^{-1}$ and so the graph $\mathrm{G}(\varphi)$ is symmetric, and that $v + \varphi(v) < 1$ for all $v \in (0,1)$.

Our mixed strategy equilibrium is Markovian with state space $\mathrm{G}(\varphi)$, and an arbitrary initial state in $\mathrm{G}(\varphi)$. At any $v \in \mathrm{G}(\varphi)$, each player i chooses a proposal $\bar{x}_i(v)$ and randomly chooses an elapse time from an exponential distribution with parameter $\lambda_i(v)$. If player 1's offer x_1 prevails, say, where possibly $x_1 \neq \bar{x}_1(v)$ (if player 1 deviates), then player 2 accepts it with fixed probability $\alpha \in (0,1)$ if $x_1 \geq \bar{x}_1(v)$, and rejects it otherwise. If the offer is rejected, the state moves to $(\varphi(x_1), x_1)$ if $x_1 \geq v_2$, and remains at v otherwise.

The functions λ_j and $\bar{x}_j$ $(j = 1,2)$ satisfy $v_i = (1-\alpha)\varphi(\bar{x}_i(v)) + \alpha(1-\bar{x}_i(v))$. Hence, player i is indifferent about making the (equilibrium) offer $\bar{x}_i(v)$ or not, and

$$v_i = \int_0^\infty \lambda_j(v) e^{-[\lambda_j(v)+r_i]t} \bar{x}_j(v) dt = \frac{\lambda_j(v)}{\lambda_j(v) + r_i} \bar{x}_j(v), \tag{1}$$

so that player i is indifferent about proposing at any moment or waiting for his opponent to propose instead. It is easy to check that $v_i < (1-\alpha)\varphi(\bar{x}_i(v)) + \alpha(1-\bar{x}_i(v))$ when $x_i > \bar{x}_i(v)$;

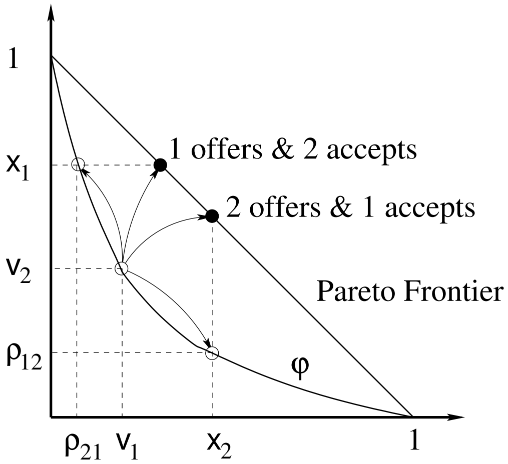

Figure 1: **Example Aspiration Set, Payoff Frontier, and Transitions.** Aspiration value pairs are denoted by ○, and agreements by ●. Lines indicate continuations following proposals. Proposals are sometimes turned down by his opponent who is indifferent.

thus, player i does not want to make a disequilibrium offer. Notice that since $\bar{x}_i(v) > v_j$, the equilibrium offer is acceptable. Also, $\lambda_i(v) < \infty$ everywhere; thus, deadlocks almost surely do not occur. This also will be a general property of our aspirational equilibria.

Between offer events, the players engage in a waiting game to see who will make the next offer. Eventually, one player breaks down and proposes a pie split; that offer may be rejected. The bargaining state randomly transitions through $\mathrm{G}(\varphi)$ as offers are made and rejected until absorption on the Pareto frontier, when an offer is finally accepted.

3 The Aspirational Refinement

Any pie split at any time is an SPE of our continuous-time model. We now introduce an equilibrium refinement motivated by the earlier psychological study of Siegel and Fouraker (1960). We assume that players' bargaining aspirations govern their behaviour. We proceed via assumptions that ensure time-constant strategies and stationarity in payoffs.

The first three properties of an SPE that we assume here are:

A1. *Time Stationarity*: Players mixed (behavior) strategies are independent of the elapse time since the last offer (or the time since zero).

This precludes time as a coordination device: Player i's expected value remains constant as long as it is possible that j makes an offer (the support restriction). We now make an essentially non-triviality assumption that is only needed to rule out certain boring corner equilibria.

A2. *Serious offers and acceptances*: Equilibrium offers are accepted with strictly positive probability. No player i ever accepts the offer $x_j = 0$, and is never the first to make a full concession offer $x_i = 1$.

This bounds away acceptance rates and offers from the boundaries.[1] Assuming serious offers A2 precludes offers as either cheap talk, payoff-irrelevant babbling, or equilibrium protocol. It cannot be common knowledge, for instance, that the first offer must be made and ignored. With our assumption in force, every offer is serious, and we are later able to make falsifiable predictions based on the offers made.

Lemma 1 *Let σ be an SPE satisfying* A1–A2. *Then for all offer nodes h, the mixture over offer times has an exponential distribution, say $F_{\lambda_i}(\tau) = 1 - e^{-\lambda_i \tau}$, where $\lambda_i \in [0, \infty]$.*

3.1 Bargaining as a Continuous Time Markov Process

We next impose that after any event history, the players' behaviour depends exclusively on their current expected values. This is a standard refinement, used in APS86 and sequels.

A3. *Action-payoff stationarity*: $\pi(\sigma|_h) = \pi(\sigma|_{h'}) \Rightarrow \sigma|_h = \sigma|_{h'}$, for all histories h, h'.

Action-payoff stationarity is the primary basis for our aspirational refinement of SPE. It asserts that the players' strategies are functions of their aspirations while bargaining. Further, as soon as a player, say 2, hears an IR offer x_1, the past is forgotten, and his behaviour only depends on the new *interim* aspiration value vector (v_1, x_1).

We now show that an SPE obeying A1–A4 admits a simple Markovian structure, summarized by a state space A, and a quintuple $(v^0, \lambda, \mu, \alpha, \rho)$, where

- $v^0 \in \mathrm{A} \equiv \{(v_1, v_2) \in \mathbb{R}^2_+ \mid v_1 + v_2 \leq 1\}$
- $\lambda = (\lambda_1, \lambda_2)$ with $\lambda_i : \mathrm{A} \mapsto \mathbb{R}_+ \cup \{\infty\}$
- $\mu = (\mu_1, \mu_2)$ with $\mu_i : \mathrm{A} \mapsto \Delta([0, 1])$

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[1] It does not rule out the possibility that the offer $x_i = 1$ be required in equilibrium after some history h. But, if $x_i = 1$ is required, it must be that along h, player i has already made the offer $x_i = 1$ (out of equilibrium) at some point.

- $\alpha = (\alpha_1, \alpha_2)$ with $\alpha_i : \mathrm{A} \times [0,1] \mapsto [0,1]$
- $\rho = (\rho_1, \rho_2)$ with $\rho_i = (\rho_{i1}, \rho_{i2}) : \mathrm{A} \times [0,1] \mapsto \mathrm{A}$

and where $\mu_i(B|v) \equiv \mu_i(v)(B)$ is a measurable function of v for each $B \in \mathrm{B}([0,1])$.

Such a quintuple intuitively recursively specifies an *aspirational profile* (AP), say $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$, as follows: Starting at stage with the initial state $v^0 \in \mathrm{A}$, each player i randomly and independently chooses a time t_i from the distribution $F_{\lambda_i(v^n)}$ (possibly ∞) to propose an offer x_i from the distribution $\mu_i(\cdot|v^n)$. If $t_1 \leq t_2$, then player 1's offer x_1 is made, and player 2 accepts it with chance $\alpha_2(v^n, x_1)$. If $t_1 > t_2$, then player 2's offer x_2 is made, and player 1 accepts it with chance $\alpha_1(v^n, x_2)$. If player i rejects the offer x_j (possibly not in $\mathrm{supp}(\mu_j(v^n))$) then $v^{n+1} = \rho_i(v^n, x_j)$. Finally, increment n by 1.

An AP $\sigma = \bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ has a simple recursive structure which we now exploit. Let $\bar{\mathcal{H}}_N \equiv \{h \in \mathcal{H}_N \mid h \text{ is finite}\}$. After any history $h \in \bar{\mathcal{H}}_N$, the continuation strategy profile $\sigma|_h$ is another AP $\bar{\sigma}(v, \lambda, \mu, \alpha, \rho)$ with the same structure but a different initial state $v \in \mathrm{A}$. So any history $h = \{(i_1, x_1, t_1), (j_1, N, t_1), \ldots, (i_n, x_n, t_n), (j_n, N, t_n)\} \in \bar{\mathcal{H}}_N$, with rejected offers at times $t_1 \leq t_2 \leq \cdots \leq t_n$, generates a state vector $V_\rho(h|v^0)$ by repeatedly applying the function ρ. Let $v^{\ell+1} = \rho_j(v^\ell, x_\ell)$, where j is i_ℓ's opponent, $\ell = 0, \ldots, n-1$; then $V_\rho(h|v^0) = v^n$. Moreover, as noted above, $\sigma|_h = \bar{\sigma}(V_\rho(h|v^0), \lambda, \mu, \alpha, \rho)$.

Definitions A strategy profile σ *coincides* with an AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ if for any offer node h, $\sigma|_h = \bar{\sigma}(\pi(\sigma|_h), \lambda, \mu, \alpha, \rho)$. An *aspirational equilibrium* (AE) is an SPE σ that coincides with an AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$.

Observe that if σ coincides with an AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ then after any offer node h, the state of the AP coincides with the value of $\sigma|_h$.

Theorem 1 *Let σ be an SPE satisfying* A1–A3. *Then σ is an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$.*

3.2 IC Condition Characterization of Aspirational Equilibrium

For any strategy profile σ, let $\mathrm{A}(\sigma)$ denote its set of continuation values (on and off the outcome path implied by σ). Formally,

$$\mathrm{A}(\sigma) = \{\pi(\sigma|_h) \mid h \text{ is an offer node}\} \subset \mathrm{A}.$$

We now identify an intuitive weak *(mutual) individual rationality* (IR) requirement for offers — it must provide both players with nonnegative surplus over their current values.

Lemma 2 *If σ is an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$, then supp$(\mu_j(v)) \subset [v_i, 1 - v_j]$ for all $v \in \mathrm{A}(\sigma)$.*

The players essentially bargain over the remaining *surplus* $1 - v_1 - v_2$. Since, by Lemma 2, any equilibrium offer x_j must obey $v_i \leq x_j \leq 1 - v_j$ at the aspiration pair (v_1, v_2), it must concede a fraction of the surplus. The *surplus concession fraction* $\kappa_j \in [0, 1]$ obeys

$$x_j = v_i + \kappa_j (1 - v_1 - v_2). \tag{2}$$

For any AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$, we denote player i's *expected offer* by $\bar{x}_i(v) = \int_0^1 x \mu_i(v)(dx)$, for $v \in \mathrm{A}$, and the corresponding *expected surplus concession fraction* by

$$\bar{\kappa}_i(v) = \frac{\bar{x}_i(v) - v_j}{1 - v_1 - v_2}$$

We now give the *incentive compatibility* conditions for an AE.

Theorem 2 (IC Conditions) *Assume σ coincides with an AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$. Then, σ is an AE iff for all $v \in \mathrm{A}(\sigma)$ and offers $x \in [0, 1]$, v_i satisfies the* waiting IC equation (3), *as well as the* offer IC equation (4), *and the* acceptance IC equation (5) *below:*

$$v_i \geq \frac{\lambda_j(v)}{\lambda_j(v) + r_i} \bar{x}_j(v) \text{ with equality if } \lambda_i(v) < \infty \tag{3}$$

$$v_j \geq \alpha_i(v, x)(1 - x) + (1 - \alpha_i(v, x)) \rho_{ij}(v, x) \text{ with equality if } x \in \mathit{supp}(\mu_j(v)) \tag{4}$$

$$\rho_{ii}(v, x) \begin{cases} = x & \text{if } \alpha_i(v, x) \in (0, 1) \\ \leq x & \text{if } \alpha_i(v, x) = 1 \\ \geq x & \text{if } \alpha_i(v, x) = 0. \end{cases} \tag{5}$$

Proof: Let h be an offer node such that $v = \pi(\sigma|_h)$ satisfies $\lambda_i(v) < \infty$ and $\lambda_j(v) > 0$. After any such history, the equilibrium conditions are equivalent to (a) player i is willing to wait for j's offer; (b) player j is indifferent among his equilibrium offers, and cannot improve his expected value by deviating; (c) after rejecting x, player i expects at least x if he is required to reject x with positive probability, and no more than x if he is required to accept x with positive probability. Condition (a) was the original logic behind (1), which yields (3) with equality. Next, any offer x is accepted by j with probability $\alpha_j(v,x)$, and when it is rejected, i's aspiration value drops to $\rho_{ji}(v,x)$. Condition (b) is equivalent to (4). Finally, (5) follows from (c), where $\rho_{ii}(v,x) > x$ only if x is an off-path offer.

Next assume $\lambda_i(v) = \infty$, so that player i makes an offer immediately. In this case, he need not be willing to wait and (3) may hold with strict inequality. Similarly, if $\lambda_j(v) = 0$, player j makes no offers (even though by definition of σ he still must choose an offer for time ∞). In this case, (4) need not be satisfied with equality when $x \in \text{supp}(\mu_j(v))$. $\square$

Suppose that $\lambda_1(v), \lambda_2(v) < \infty$ for all $v \in \mathrm{A}(\sigma)$. Then, $\bar{x}_i(v) > v_1$ by (3), while (4) asserts that i's expected continuation value upon offering is his current value v_i. In other words, the payoff to waiting exceeds that of offering. Players are thus engaged in a sequence of *wars of attrition* until agreement. Put differently, there is an endogenous *proposee advantage*, unlike the hard-wired proposer advantage with temporal monopoly.

It is easy to construct AE's using Theorem 2. We will build on the earlier symmetric aspiration function $\varphi(x) = (1-x)/(1+x)$. Observe that even with pure offers, the IC conditions allow two degrees of freedom for each player's choice of $(\lambda, \mu, \alpha, \rho)$. So fixing the aspiration set (and thereby ρ) still leaves one degree of freedom in choosing (λ, μ, α). Earlier, we also assumed a constant acceptance rate α. We now consider a constant surplus concession. In section 3.3, we shall see that a constant offer rate λ is impossible.

Example (Constant Surplus Concession): In a *κ-concession rule*, each player concedes a fixed surplus fraction $\kappa \in (0,1]$ whenever he makes an offer. By (3) and (4),

$$\lambda_i(v) = \frac{r_j v_j}{\kappa(1 - v_1 - v_2)} \quad \text{and} \quad \alpha_j(v,x) = \frac{v_i - \varphi(x)}{1 - x - \varphi(x)}.$$

(The definition of $\alpha_j(v,x)$ can be modified to strictly punish player i for deviant offers, if so desired.) Then one can easily verify that $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ is an AE for all $v^0 \in \mathrm{G}(\varphi)$.

Finally, an AE cannot have both a constant acceptance rate *and* constant conceded surplus fraction κ. For a constant κ is equivalent to a constant angle of incline from the aspiration vector v to the offer $(1 - \bar{x}_1, \bar{x}_1)$. A constant acceptance rate then forces the aspiration set $\mathrm{A}(\sigma)$ to lie inside and parallel to the Pareto frontier. This contradicts Lemma 6 below, that $\mathrm{A}(\sigma)$ must approach the Pareto frontier at the extremes.

We can now rigorously establish one of our claimed 'intuitive' properties of an AE.

Corollary 1 *If j's equilibrium offer* x_j *is rejected with strictly positive chance, and rejection incurs more delay, then rejection strictly harms j, and acceptance strictly helps j.*

Proof: Let $v \in \mathrm{A}(\sigma)$ be the current value of the AE σ. Delay after rejection implies $\rho_{ii}(v, x_j) + \rho_{ij}(v, x_j) < 1$. By assumption, $0 < \alpha_i(v) < 1$, and therefore $\rho_{ii}(v, x_j) = x_j$ and $\rho_{ij}(v, x_j) < 1 - \rho_{ii}(v, x_j) = 1 - x_j$. Finally, by (4), v_i is the strict convex combination $\alpha_i(v, x_j)(1 - x_j) + (1 - \alpha(v, x_j))\rho_{ij}(v, x_j)$, and thus $\rho_{ij}(v, x_j) < v_j < 1 - x_j$. □

3.3 The Proposal Rate

A. Infinite or Zero Proposal Rates. The element of our model absent from temporal monopoly models is the offer rate. We first must understand when player i never offers, or insistently does so — i.e. when the rate λ_i implied by an AE is zero or infinity.

Lemma 3 *Assume* σ *is an AE* $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$*. For any* $v \in \mathrm{A}(\sigma)$*: (a)* $\lambda_i(v) = 0$ *implies that* $\lambda_j(v) > 0$*; (b)* $\lambda_i(v) = \infty$ *for some player i iff* $v_1 + v_2 = 1$*, in which case the game ends immediately with the split v.*

If $v_1^0 + v_2^0 < 1$, the state vector v is never on the Pareto frontier (until agreement), and hence, infinite offer rates are never observed on the equilibrium path. But the state v can land on the Pareto frontier after an off-path offer. For example, player i may make an out of equilibrium offer, which calls for player j to reject with probability 1. After rejection, the continuation value vector is arbitrary and can be chosen to be on the Pareto frontier.

We now address the opposite case where $\lambda_i = \infty$, which forces $v_1 + v_2 = 1$.

Corollary 2 *Immediate agreement on any efficient* v *with* $v_1, v_2 \in (0, 1)$ *is an AE outcome.*

In light of this corollary, the interesting implications obtain for AE with delay. We thus *hereafter assume* $v_1^0 + v_2^0 < 1$*,* and so we have delayed agreement, as $\lambda_1, \lambda_2 < \infty$.

B. Exploding Proposal Rates on the Boundary. We now consider the boundary behavior of the proposal rates λ_i. Work by Perry and Reny (1993), Sakovics (1993), and Stahl (1993) showed that imposing a boundedly positive delay time between offers confers an offerer advantage — for then declining burns a boundedly positive fraction of the pie. It's instructive to see that this is not a feature of our model. Indeed, offer rates must explode in some subgames. To see this, let $\mathrm{A}_{IR}(\sigma) \subset \mathrm{A}(\sigma)$ be all value vectors that can be reached after histories with only IR offers.[2] We argue that $\mathrm{A}_{IR}(\sigma)$ touches or approaches the Pareto frontier, and that the proposal rate blows up at these aspiration vectors.

Theorem 3 (Boundary Proposal Rates) *Let σ be an AE $\bar{\sigma}(v^0, \lambda, \pi, \alpha, \rho)$. There exist boundary aspiration vectors $w = (y, 1-y)$, where $y \in (0,1)$, either on the aspiration set, or as a limit point. At each such aspiration vector, $\lim_{v \to w} \lambda_i(v) = \infty$, $i = 1, 2$. If $y = 0$, then $\lim_{v \to w} \lambda_1(v) = \infty$, and if $y = 1$, then $\lim_{v \to w} \lambda_2(v) = \infty$.*

Proof: The existence of such boundary aspiration vectors is proven in the appendix. The rest of the proof is illustrative of the simplicity of an AE. Indeed, since $\mathrm{supp}(\mu_1(v)) \subset [v_2, 1 - v_1]$, as $v \in \mathrm{A}_{IR}(\sigma)$ approaches $w = (y, 1-y)$, $\mu_1(v)$ converges in probability to a point mass at $1 - y$. By condition (3) of an AE (in Theorem 2)

$$1 - y = \lim_{v \to w} \bar{x}_1(v) = \lim_{v \to w} \left[1 + \frac{r_2}{\lambda_1(v)}\right] v_2.$$

If $1 - y \neq 0$, then $\lim_{v \to w} \lambda_1(v) = +\infty$. Similarly, (3) yields $\lim_{v \to w} \lambda_2(v) = +\infty$ if $y \neq 0$ □

That is, as a player's aspiration value converges to the lowest possible payoff he will ever get in any AE, he makes offers at an increasingly unbounded pace. If $y \in (0,1)$, when the aspiration vector v is near the Pareto frontier and the surplus $1 - v_1 - v_2$ is close to 0, then both players make offers very often. When $y = 0$ (resp. $y = 1$), the conclusion only applies to player 1 (resp. 2). Recall that for the AE σ specified by a κ-concession rule and the function $\varphi(x) = [1 - x]/[1 + x]$, we have that $\lambda_i(v) = r_j/[\kappa v_i]$. Thus, this σ provides an example where $\lambda_2(v)$ but *not* $\lambda_1(v)$ tends to ∞ as $v \to (1, 0)$.

EXAMPLE (BATTLE OF THE SEXES): Suppose that in equilibrium, players are engaged in a war of attrition to first propose their less favoured pie split among $(2/3, 1/3)$ and $(1/3, 2/3)$; such an offer is always accepted. (Think of this as a method of deciding which pure equilibrium to play in a Battle of the Sexes.) This is an AE with $v^0 = (1/3, 1/3)$ provided $\lambda_1 = r_2$ and $\lambda_2 = r_1$ (to satisfy the IC conditions). But $\mathrm{A}(\sigma)$ must contain more

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[2] The aspiration value set $\mathrm{A}(\sigma)$ includes values that can only be reached after deviations from σ. For much of our analysis, only aspiration values in $\mathrm{A}_{IR}(\sigma)$ that can be reached on the equilibrium path matter. Accordingly, $\mathrm{A}_{IR}(\sigma)$ includes all equilibrium values and possibly some non equilibrium values, but it excludes, for example, values reachable only if one player's overly generous offer were rejected.

than just v^0, as player i can always make offers like $x = 1/2$. The simplest aspiration set is $\mathrm{A}(\sigma) = \{(1/3, 2/3), (1/3, 1/3), (2/3, 1/3)\}$ — where after any offer by i, the value reverts at once to j's favoured outcome. For $v \in \{(1/3, 2/3), (2/3, 1/3)\}$, at least one offer rate must be infinite for immediate agreement.

Notice that the conclusion of Corollary 1 fails for this example. In fact, if the offer is rejected (off-path), then agreement is still immediate at that proposal. No harm is done.

In the appendix, we show by example that the proposal rate can also explode in the middle of the aspiration set if individual offers can concede arbitrarily little surplus.

4 Properties of Aspirational Equilibrium

4.1 Bargaining Duration via Aspirations as a Submartingale

Denote by $\tau(v^0)$ the random time to agreement when the initial aspiration vector is $v(0) = v^0$. When the players follow an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$, at any time t until $\tau(v^0)$, each player has a well-defined aspiration level $v_i(t)$ between offer events. Moreover, if an offer transpires at time t, then let $v_i(t)$ be the expected value of player i immediately after the proposal is tendered, but before its content is heard, and $\bar{v}_i(t)$ just after the content is heard and the reply is given. (Define $\bar{v}_i(t) = v_i(t)$ at all other times.)

We now assert that the process $e^{-r_i t} v_i(t)$ is a martingale 'until' the players reach an agreement at time $\tau(v^0)$. We eliminate the termination date in the following artificial stochastic process, where we "freeze" the terminal value:

$$z_i(t) = \begin{cases} e^{-r_i t} v_i(t) & \text{for } t \leq \tau(v^0) \\ e^{-r_i \tau(v^0)} \bar{v}_i(\tau(v^0)) & \text{for } t > \tau(v^0). \end{cases}$$

The process $\bar{z}_i(t)$ is likewise defined, using $\bar{v}_i(t)$ for $t \leq \tau(v^0)$ instead.

Theorem 4 *Let σ be an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$. The stochastic processes $z_i(t)$ and $\bar{z}_i(t) = e^{-r_i t} v_i(t)$ are martingales. The aspiration process $v_i(t)$ is a strict submartingale until agreement time τ.*

For an intuition about the martingale, either player is indifferent about offering, since he expects his value back by waiting and never offering. Of course, the final agreement time may owe to either player proposing, and so conditional on getting proposed to, the expected discounted value exceeds the current value. The proof shows that this surplus over the martingale exactly balances the loss in the event that a player ends up as proposer.

By the martingale convergence theorem, since values are a bounded submartingale, they almost surely converge. But being generated by an AP, they are also a Markov process; so any limit value v must be stationary under the dynamic. But the only stationary values are on the Pareto frontier, where the real delay time is zero, by Lemma 3. Hence,

Corollary 3 (Bargaining Finiteness) *Bargaining ends almost surely in finite time.*

Not only is the bargaining duration $\tau(v^0)$ a.s. finite, but we can bound it below too:

Corollary 4 (*a*) *In an AE, we have* $E[\tau(v^0)] \geq -\log[v_1^0 + v_2^0]/\max(r_1, r_2)$. *Hence,* (*b*) *if players i and j made the the last two offers, x_i and x_j, and both were rejected, then the expected time until the bargaining ends is at least* $-\log(x_1 + x_2)/\max(r_1, r_2)$.

Proof: Denote $\bar{r} = \max(r_1, r_2)$ and $\bar{\tau} = E[\tau(v^0)]$. By the Optional Stopping Theorem,

$$v_i^0 = E[\bar{z}_i(\tau(v^0))|\bar{z}_i(0)=v_i^0] = E[e^{-r_i\tau(v^0)}\bar{v}_i(\tau(v^0))|v_i^0] \geq E[e^{-\bar{r}\tau(v^0)}\bar{v}_i(\tau(v^0))|v_i^0]$$

since $\bar{z}_i(t)$ is a martingale process, and $r_i \leq \bar{r}$. Because $(\bar{v}_1(\tau(v^0)), \bar{v}_2(\tau(v^0)))$ is the final agreed pie split, it lies on the Pareto frontier, or $\bar{v}_1(\tau(v^0)) + \bar{v}_2(\tau(v^0)) = 1$. Finally, since $E[e^{-\bar{r}\tau(v^0)}] \geq e^{-\bar{r}\bar{\tau}(v^0)}$ by Jensen's inequality, we have $v_1^0 + v_2^0 \geq e^{-\bar{r}\bar{\tau}}$, as required in (*a*).

For part (*b*), after the first offer is rejected, $v_2 = x_1$. After the second offer, $v_1 = x_2$ and v_2 drops below x_1 (Corollary 1). Thus, $v_1 + v_2 \leq x_1 + x_2$. Now apply part (*a*). $\square$

4.2 Relating Offer Rates, Offers, and Acceptance Rates

We now relate our three strategic choices (λ, μ, α) — suppressing arguments where clear.

A. Offers and Timing. The war of attrition aspect of bargaining yields a simple testable implication about rejected offers alone, consistent with Siegel and Fouraker (1960).

Corollary 5 (Consecutive Offers) *If a player makes consecutive equilibrium offers, then his second offer is strictly more generous to the other player. If players i, j make sequential offers x_i and x_j, then player j offers i less than i offered himself:* $x_j \leq 1 - x_i$.

Proof: After player i makes a proposal that is rejected, the new aspiration value shifts down for player i, and up to x_i for player j. Now apply Lemma 2. $\square$

The IC conditions yield stronger implications about offer timing and acceptance rates. By the new delay IC (10):

$$v_i = \frac{r_j \lambda_j \bar{\kappa}_j}{r_i r_j + r_i \lambda_i \bar{\kappa}_i + r_j \lambda_j \bar{\kappa}_j} \tag{6}$$

From this formula, as the chance that j offers vanishes, i's value vanishes. By contrast, in a temporal monopoly, as the chance that j gets to offer vanishes, j's value vanishes.

Equation (6) also betrays a crucial separability between the delay and offer IC equations (10) and (4), evident in our examples and theorems. Surplus concessions κ interact with offer rates λ alone to determine current values in (10); the aspiration set is only relevant in the linkage in (4) between surplus concessions κ and acceptance rates α.

Corollary 6 Ceteris paribus, *the rate* λ_j *that player* j *offers varies inversely with the expected fraction of surplus in the pie* $\bar{\kappa}_j$ *that he concedes.*

Corollary 6 captures the inherent trade-off between offer timing and content: *Ceteris paribus* (holding values fixed), if a player anticipates a more generous offer, then he should expect to wait longer. This is consistent with Corollary 4, that less generous alternating offers foretell a greater lower bound on the expected bargaining duration.

Equation (6) also provides insights into the nature of bargaining power. With temporal monopoly, there are two exogenous sources of asymmetry: relative impatience levels, and the offering order. In our aspirational paradigm, offering order is not an issue, but there is a intuitive new strategic component of bargaining power: Parties gain strength from their *refusal* to make offers. As is so often true in social bargaining, one can hurt the other party by according him the "silent treatment", forcing him to make all overtures.

B. Offers and Acceptance Rates. Our theory is much richer when we move beyond AE like the Battle of Sexes where players simply "make an offer that can't be refused." We now relate the content of offers to the acceptance rates. By using coarse bounds from the IC conditions, we find two necessary inequalities jointly satisfied by all parameters $(\lambda, \kappa, \alpha)$. They are especially useful as they obtain *for any aspiration set,* and any offer.

Corollary 7 *Consider an offer* x_i *by player* i *that might be rejected* $(\alpha_j < 1)$. *Then*

$$\frac{\alpha_j}{1-\alpha_j}(1-\kappa_i) r_i < \lambda_j \bar{\kappa}_j \tag{7}$$

Proof: Using (2), rewrite the offer IC equation (4) as

$$(1-\alpha_j)(v_i-\rho_{ji}(v,x_j)) \geq \alpha_j(1-\kappa_i)(1-v_1-v_2)$$

As rejection values are positive (by Lemma 1), and $\alpha_j < 1$, (10) yields the inequality. □

Corollary 7 captures the trade-off between the acceptance rate and surplus concession of a given offer: *Ceteris paribus,* if an offer concedes too little surplus, it must be accepted with a small chance. Corollary 7 shows that incentive constraints inherently force lower bounds on surplus concession fractions and offer rates, and upper bounds on acceptance rates. Eg., if offers are accepted at least half the time, and concede half the surplus ($\kappa_1 = \kappa_2 = 1/2$), then i's proposal rate must exceed j's interest rate. Only in extreme cases like the Battle of Sexes, which concede all the surplus, can offers almost surely be accepted.

4.3 Simple Aspiration Sets

(under construction, as we are rethinking the assumptions)

A. Decreasing Aspiration Sets. We now give plausible assumptions that guarantee a *continuously decreasing* IR aspiration set $\mathrm{A}_{IR}(\sigma)$: There exists an open interval $W = (\underline{w}, \bar{w}) \subset [0,1]$ (with closure $\bar{W} = [\underline{w}, \bar{w}]$) and a continuous, strictly decreasing function $\varphi : \bar{W} \to [0,1]$ with: $\varphi(\underline{w}) = 1-\underline{w}$, $\varphi(\bar{w}) = 1-\bar{w}$, and $\mathrm{A}_{IR}(\sigma) \cup \{(\underline{w}, 1-\underline{w}), (\bar{w}, 1-\bar{w})\} = \mathrm{G}(\varphi)$. Depending on σ, $(\underline{w}, 1-\underline{w})$ and $(\bar{w}, 1-\bar{w})$ may or may not be in $\mathrm{G}(\varphi)$.[3] The simplicity of the constant α and κ examples owed to this property of the aspiration sets.

We next assume in B1 that the same offer to a player with a greater value be accepted with a lower chance; we also assume away in B2 examples like the Battle of Sexes. Not conceding all surplus is in the same spirit as the second part of A2.

B1. *Strict Acceptance Monotonicity.* Fix v_j. The chance $\alpha_i(v, x_j)$ that i accepts j's offer x_j is strictly decreasing in the proposee's value v_i for all $v \in \mathrm{A}$ and $x_j \in [v_i, 1-v_j]$.

B2. *No Full Concessions.* For every $v \in \mathrm{A}_{IR}(\sigma)$, $\bar{\kappa}_i(v) < 1$, $i = 1, 2$.

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[3] If $(1-x, x) \in \mathrm{A}_{IR}(\sigma)$ were reached from $v \in \mathrm{A}_{IR}(\sigma)$ after player 1, say, made an offer x and player 2 rejected, then $w \leq (1-x, x)$ (because the offer x is IR). That is, $1-x = \rho_{21}(v, x)$. By (4), $v_1 = 1-x$, for otherwise offering x strictly increases player 1's expected value, contradicting B4 below. Hence, $(1-x, x) \in \mathrm{A}_{IR}(\sigma)$ can only be reached in the limit after a cluster point. Thus, $(\underline{w}, \varphi(\underline{w})) \in \mathrm{A}_{IR}(\sigma)$ only if σ can produce a cluster there.

Assumptions $B1$–$B2$ are satisfied by the constant κ and α examples (φ being 1-1). They should be viewed in the same light as the earlier assumption on serious offers, as ruling out boundary cases.

Lemma 4 *Given assumptions $B1$–$B2$, the IR aspiration set $\mathrm{A}_{IR}(\sigma)$ is decreasing.*

Suppose that a mixture over offers is entertained. Casual empiricism suggests that the reason a proposer might consider sweetening an offer is to ensure a greater chance of acceptance. In his classic text, eg., Zeuthen (1930) simply assumes it as a behavioural postulate of bargaining. In fact, this fails in our context unless the aspiration set is well-behaved: Perhaps a less generous offer is rewarded by a better continuation value.

Corollary 8 *Assume $B1$–$B2$ (so that $\mathrm{A}_{IR}(\sigma)$ is continuously decreasing). If $x' > x$ are two IR offers made by player i at the value $v \in \mathrm{A}_{IR}(\sigma)$, then $\alpha_j(v, x') > \alpha_j(v, x)$.*

Proof: We have $v_i = \alpha_j(v, z)(1 - z) + (1 - \alpha_j(v, z))\rho_{ji}(v, z)$ for $z = x$ and $z = x'$. Since $\mathrm{A}(\sigma)$ is decreasing, $\rho_{ji}(v, x) > \rho_{ji}(v, x')$, and $\alpha_j(v, x) < \alpha(v, x')$. □

We can finally establish another of our claimed intuitive properties of an AE: strictly IR offers. Corollary 7, without assumptions $B1$–$B2$, directly implies that the expected offer is strictly IR: $\bar{\kappa} > 0$. Using $\mathrm{A}_{IR}(\sigma)$ decreasing, we now deduce this for all AE offers.

Corollary 9 *Assume $B1$–$B2$. If $v \in \mathrm{A}_{IR}(\sigma)$ with $v_1 + v_2 < 1$ and x_i is an AE offer at v, then x_i strictly increases player j's aspiration: $x_i > v_j$.*

Proof: By Lemma 2, suppose instead $x_i = v_j$. Since $\mathrm{A}_{IR}(\sigma)$ is decreasing, player i gets $1 - x_i > v_i$ if the offer is accepted (which happens with positive chance by A3) and v_i otherwise. This yields player i an immediate positive expected gain. Contradiction. □

B. Convex Rejections and Consecutive Offers. We introduce a stronger property of the IR aspiration set: $\mathrm{A}_{IR}(\sigma)$ has *convex rejections* if for all $v \in \mathrm{A}_{IR}(\sigma)$ and IR offers x_1, x_2,

$$\frac{1 - \rho_{12}(v, x_2)}{\rho_{11}(v, x_2)} < \frac{1 - v_2}{v_1} < \frac{1 - \rho_{22}(v, x_1)}{\rho_{21}(v, x_1)} \tag{8}$$

This property obtains if φ is a convex function, and was satisfied by our constant α and κ examples. It will permit more powerful and intuitive conclusions about consecutive offers.

Lemma 5 (Convexity) *The aspiration set* $\mathrm{A}_{IR}(\sigma)$ *has convex rejections, iff for* $i = 1, 2$*,*

$$\frac{\alpha_j}{1-\alpha_j} > \frac{\kappa_i}{1-\kappa_i} \cdot \frac{\lambda_j \bar{\kappa}_j}{r_i + \lambda_j \bar{\kappa}_j} \tag{9}$$

In particular, it suffices that $\alpha_1 > \kappa_2$ *and* $\alpha_2 > \kappa_1$*.*

Proof: Using (7) and then (6), the right inequality of (8) holds iff

$$1 - \frac{\alpha_2(1-\kappa_1)r_1}{(1-\alpha_2)\lambda_2\bar{\kappa}_2} \quad = \quad \frac{\rho_{21}(v, x_2)}{v_1} < \frac{1-\rho_{22}(v, x_1)}{1-v_2} = 1 - \frac{\kappa_1 r_1 r_2}{r_1 r_2 + r_2 \lambda_2 \bar{\kappa}_2}$$

since $\rho_{22}(v, x_1) = x_1 = v_2 - \kappa_1(1 - v_1 - v_2)$ by (2). This yields (9) for $i = 1$. □

So the convex rejections property obtains provided players' offers do not concede too much surplus, or are not too likely to be rejected. Lemma 5 and (10) imply:

Corollary 10 (Consecutive Offers, Encore) *Assume* (9)*, or more simply* $\alpha_1 > \kappa_2$ *and* $\alpha_2 > \kappa_1$ *(so that* $\mathrm{A}_{IR}(\sigma)$ *has convex rejections). Then* $\lambda_j \bar{\kappa}_j$ *rises after player* j*'s proposal is rejected; that is, he either must concede more surplus or offer more rapidly.*

Suppose that player j's offer has been rejected. We knew from Corollary 5 that his next offer must be more generous. We now know that $\lambda_j \bar{\kappa}_j$ is lower: He expects a smaller share of the surplus. He must now either accelerate his proposal rate, or concede more surplus.

5 Sensitivity Analysis

We finally study what happens if one player becomes more impatient. As our bargaining model admits a multiplicity of AE, it is not immediately obvious how to perform such an exercise. Indeed, the AE played may vary with the interest rates (r_1, r_2). We first observe that for *fixed concessions and timing* of both players, any player i's value decreases with his relative impatience r_i/r_j, by (10). But this says nothing about the effect on the *ultimate pie split*. Perhaps more impatient individuals tend to enjoy smaller bargaining delays. Our simple approach below pursues this very line of thought, focusing on the proposal rate.

Notice that $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ is an AE for the interest rates $r = (r_1, r_2)$ iff $\bar{\sigma}(v^0, \hat{\lambda}, \mu, \alpha, \rho)$ is an AE for $\hat{r} = (\hat{r}_1, \hat{r}_2)$, where $\hat{\lambda}$ are the corresponding offer rates $\hat{\lambda}_i = (\hat{r}_j/r_j)\lambda_i$, $i = 1, 2$. So given any AE σ, if player j becomes twice as impatient, and i doubles the rate that he offers in σ, then the new strategy $\hat{\sigma}$ is an AE. This simple mapping, adjusting the offers rates in response to an interest rate change, but otherwise fixing the strategy profile and aspiration set, allows us to compare the *sets of AE* for different levels of impatience by just comparing corresponding equilibria.[4] We say that *player i's expected pie split is higher* with $(\hat{r}_1, \hat{r}_2)$ than with (r_1, r_2) if the expected final split in each AE $\hat{\sigma} = \bar{\sigma}(v^0, \hat{\lambda}, \mu, \alpha, \rho)$ for $(\hat{r}_1, \hat{r}_2)$ exceeds that of the corresponding AE $\sigma = \bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ for (r_1, r_2).

For our next result, we first need an enhanced war of attrition property:

B3. *Strong War of Attrition.* For any $v \in \mathrm{A}_{IR}(\sigma)$, expected offers obey $\bar{x}_1(v) + \bar{x}_2(v) \geq 1$.

As previously noted, the players are engaged in a war of attrition in terms of expected continuation values; assumption B3 requires that the war of attrition property holds (weakly) in terms of the immediate expected offers (i.e. ignoring rejection values). Written in the form $\bar{x}_i(v) \geq 1 - \bar{x}_j(v)$, it has the following interpretation. At any $v \in \mathrm{A}_{IR}(\sigma)$, the offer by i to j is more generous to j than what j would offer himself. That is, in terms of the next offer alone, it is always better to let your opponent speak first.

The next sensitivity analysis result is proven in the appendix.

Theorem 5 (Impatience Helps) *Let σ be an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ satisfying B1–B3. If $\hat{r}_i/r_i > \hat{r}_j/r_j$, then player i's expected pie split is higher with interest rates $\hat{r}$ than with r.*

6 Conclusion

Summary. We have analyzed the complete information bargaining problem truly without procedures. Our approach therefore restricts the equilibrium concept rather than the action space. Our refinement of SPE is behaviourally-based, and inspired by research on the importance of players' aspirations in decision-making. It also yields a very tractable theory of bargaining. While not unique, all AE have the same intuitive elements of bargaining absent from the temporal monopoly theory: wars of attrition endogenously arise; offers are concessions; offers may be declined, disappointing the proposer, or accepted,

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[4] Milgrom and Roberts (1994) perform a similar comparison exercise for a monotone environment, using the strong set order; this order, for instance, yields unambiguous shifts in the infimum and supremum of the equilibrium set. Neither their nor our exercise can be dispositive of how the realized equilibrium actually changes. Here, one could even imagine that some more complex mapping might yield an opposite set comparison. However, we just mean to question the unambiguity of the standard comparative static.

strictly pleasing him. Offer timing and content are also entwined. For instance, an embedded martingale structure yields immediate bounds on the duration of bargaining. Further, a player's second consecutive offer must be more generous that his first. We have shown more generally how acceptance rates, surplus conceded, and offer rates are related.

Relative to temporal monopoly, our refinement inverts the strategic role of the proposer — no longer strength but weakness. This changes many results, and casts doubt on the standard impatience comparative static. We show that for a natural mapping between AE for different discount rates, increased impatience skews the final pie split in one's favour. Our methodology, showing how the expected absorbing state of a Markov process changes as the initial state changes, is new and should be applicable elsewhere.

En route, we have given a new formalization of continuous time games in the spirit of standard extensive forms, specialized to our complete information bargaining context.

Literature Reprise. Temporal monopoly has been the foundation of most if not all dynamic noncooperative bargaining papers written since 1982. Even behavioural bargaining papers have adopted this paradigm. For instance, Yildiz (2001) admits different priors over *who has the temporal monopoly* each period (the 'recognition' process). Our behaviourally-founded bargaining paper escapes this paradigm. We do, however, omit an obviously important aspect of bargaining, by avoiding incomplete information. That complication is the natural next hurdle to surmount.

This paper returns to some early influential views of bargaining. Edgeworth (1881), for instance, thought [§II, p. 51] "there are an indefinite number of arrangements à priori possible, towards one of which the system is urged by . . . the *Art of Bargaining* — higgling dodges and designing obstinacy, and other incalculable and often disreputable accidents." Zeuthen (1930) likewise argued: "Every solution is not equally probable, and if you take a large enough number of cases, you may expect the economic forces to express themselves." By avoiding temporal monopoly, we have given just such a stochastic story of bargaining, despite complete information. Our randomness instead owes to mixed strategies and imperfect information.

There is also a long-standing view of bargaining as an irreversible concession game, which has naturally led to war of attrition analyses.[5] Osborne (1985) offers a simple com-

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[5] For a striking example, the pre-Ståhl frontier bargaining textbook Coddington (1968) amazingly formalizes the bargaining problem as "represented quite generally by . . . (1) a pair of variables q_1, q_2

plete information war of attrition bargaining game, but unlike our completely unrestricted offers, assumes fixed offers.[6] Abreu and Gul (2000) is a recent incomplete information war of attrition. The war of attrition in Abreu and Pearce (2001) even owes to a concession game. Our bargaining *is* a war of attrition, and yet *is not* a concession game. To see this distinction, assume players each have two units in Abreu and Pearce's five unit bargaining problem. In their pure concession game, a player's concession of the last unit cannot be rejected. Just as in Rubinstein (1982), we have instead assumed exploding offers — i.e. implying no future commitment: Bygones are bygones.[7]

Exploding offers not only ensures an analysis that is stationary in aspiration space, but also captures the realistic inherent risk of declining any offer: One may ultimately offer or accept a strictly worse outcome. Accepting an offer is like an irreversible decision to exercise an option, and spurning one like a risky decision not to sell an asset. If offers implied irrevocable commitments, some sort of third party commitment technology would be required to enforce equilibria where the players gradually concede the pie.

A Appendix: Omitted Proofs

The proper subgame after an offer node h is formally equivalent to the original game, after resetting the clock. To flesh this out, we need some notation. Introduce a forward time-shift operator $\Upsilon_{(t,k)}$ on histories. For all $h' \in \mathcal{H}$ and vector times (τ, ℓ), let

$$\Upsilon_{(\tau,\ell)}(h') \equiv \{(i, s, (\tau, \ell) \oplus (t, k)) | (i, s, (t, k)) \in h'\}.$$

Let h be an offer node and put $(\tau, \ell) = T(h)$. We now define the history: h followed by $h' \neq \emptyset$. If $\ell = \infty$, the first event in h' must occur after a positive elapse time. Then $h \cup \Upsilon_{(\tau,\ell)}(h')$ concatenates the prior history h with the new history h'. Thus for any offer node h, and any $h' \in \mathcal{H}$, define $\sigma_i|_h(h') = \sigma_i(h \cup \Upsilon_{(\tau,\ell)}(h'))$. Then $\pi_i(\sigma|_h)$ is player i's expected continuation value after h, discounted from time $T_1(h)$.

Given an offer node h and $t > 0$, let $\pi(\sigma|h, t)$ denote the expected payoff vector of following σ after history h (discounted from time $T_1(h) + t$), given that no intervening

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representing the demands of the bargainers at any point in time." This view held at least since Hicks (1932) and Zeuthen (1930), whose concession games unfolded in one instantaneous mental flash.

[6] Our Battle of Sexes with fixed offers has this flavour for on-path, but not off-path, offers.

[7] eg. Fershtman and Seidmann (1993) assume players can't accept worse offers than they have rejected.

event has occurred after h in $[T_1(h), T_1(h)+t)$. Notice that $\pi(\sigma|h, 0) = \pi(\sigma|_h)$.

A.1 Proof of Theorem 1

Let σ be an SPE obeying A1–A3. We first define $(v^0, \lambda, \mu, \alpha, \rho)$. For each offer history h, let $v = \pi(\sigma|_h)$. Given A1–A3, Lemma 1 asserts that after h, players' offer times follow exponential distributions, with parameters $\bar{\lambda}_1, \bar{\lambda}_2$. Put $\lambda_i(v) = \bar{\lambda}_i$, for $i = 1, 2$. By assumption A1, the offer distributions $\sigma_i^x(h)$ and $\sigma_i^t(h)$ are independent; let $\mu_i(v) = \sigma_i^x(h)$. Given the atomless exponential distribution of proposal times, in equilibrium, ties and instantaneous offers almost surely do not occur (and so do not affect expectations).

Assume that after the offer history h, player i proposes first. Player j accepts an offer x_i at time $T_1(h) + t$ with chance $\alpha_j(v, x_i) = \sigma_j(h \cup \{(i, x_i, (T_1(h) + t, 1))\}) = \gamma_j(\{(i, x_i, (t, 1))\})$, where $\gamma = \sigma|_h$. By A4, γ depends on h only through $v = \pi(\sigma|_h)$, and by A1, $\gamma_j(\{(i, x_i, (t, 1))\})$ does not depend on t, so that α_j is well defined. For such an offer, define the tail history $h' = \{(i, x_i, (t, 1)), (j, N, (t, 2))\}$, and let $\rho_j(v, x_i) = \pi(\gamma|_{h'})$. Again, ρ_j is well defined because γ only depends on v.

Fix a history h, and let t^* be the last time of any cluster point or deadlock in h, if any, or set $t^* = 0$. Decompose $h = h^* \cup \Upsilon_{(t^*,1)}(h')$, where $h' \in \bar{\mathcal{H}}_N$, and let $v^* = \pi(\sigma|_{h^*})$ and $v = \pi(\sigma|_h)$. By definition, $\sigma|_h = \bar{\sigma}(v^*, \lambda, \mu, \alpha, \rho)|_{h'} = \bar{\sigma}(v, \lambda, \mu, \alpha, \rho)$ since $v = V_\rho(h'|v^*)$. Hence, $v = \pi(\bar{\sigma}(v, \lambda, \mu, \alpha, \rho))$.

A.2 Mutual Individual Rationality: Proof of Lemma 2

After an offer $x_j \in \text{supp}(\mu_j(v))$ is made, player i's continuation value is x_j, and since $\alpha_i(v, x_j) > 0$, $x_j \geq v_i$. We claim that $x_j \leq 1 - v_j$. By contradiction, assume $x_j > 1 - v_j$. If i accepts this offer, j gets $1 - x_j < v_j$. Alternatively, if $\alpha_i(v, x_j) < 1$ and i rejects this offer, then i's continuation value is $\rho_{ii}(v, x_j) = x_j$. Since $\rho_i(v, x_j) \in \mathrm{A}$, we have $\rho_{ii}(v, x_j) + \rho_{ij}(v, x_j) \leq 1$ and $\rho_{ij}(v, x_j) \leq 1 - x_j$. So j's expected value after offering x_j equals $\alpha_i(v, x_j)(1 - x_j) + (1 - \alpha_i(v, x_j))\rho_{ij}(v, x_j) < v_j$. In either case player j's continuation value is less than v_j, a contradiction. Hence, $\text{supp}(\mu_j(v)) \subset [v_i, 1 - v_j]$.

A.3 Positive Aspiration Values: Proof of Lemma 3

Claim 1 *If σ is an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ and $v \in \mathrm{A}(\sigma)$, then $v > 0$.*

Proof: (Part 1) By contradiction, suppose first that $(0, v_2) \in \mathrm{A}(\sigma)$ for some $v_2 \in (0, 1]$. If player 1 offers $x_1 = 1$ and player 2 rejects it, the continuation value is $(0, 1)$. Since $v_2 < 1$, player 1 never offered $x_1 = 1$ along the history leading to $(0, v_2)$. But starting at $(0, v_2)$, the continuation strategy must deliver the pie split $(0, 1)$. Thus, in equilibrium either 1 offers $x_1 = 1$ or 2 offers $x_2 = 0$ and player 1 (eventually) accepts it, contrary to A2.

(Part 2) Finally, we show that $(0, 0) \notin \mathrm{A}(\sigma)$. If $(0, 0) \in \mathrm{A}(\sigma)$, then starting at $v = (0, 0)$, no one proposes again. Consider the out of equilibrium offer $x_1 \in (0, 1)$ by player 1. If player 2 accepts this offer, then 1 gets $1 - x_1 > 0$, violating (4). Hence, $\alpha_2(v, x_1) = 0$. But for player 2 to be willing to reject x_1, we must have $\rho_2(v, x_1) = (0, v_2') \in \mathrm{A}(\sigma)$ for some $v_2' \geq x_1$, contradicting Part 1. $\square$

Proof: Let $v = \pi(\sigma|_h) \in \mathrm{A}(\sigma)$ for the offer node h. Assume $\lambda_i(v) = 0$. If $\lambda_j(v) = 0$, then bargaining stops and expected values are $\pi(\sigma|h) = (0, 0)$. This contradicts the Claim 1.

For (b), assume that $\lambda_i(v) = \infty$. Then after history h, player i offers x_i immediately. From Lemma 2, $\mathrm{supp}(\mu_i(v)) \subset [v_j, 1 - v_i]$. If $x_i > v_j$, then player j earns at least x_i by accepting, so that $\pi_j(\sigma|h) > v_j$. Contradiction. Hence, $\mu_i(v)$ is a point mass at $x_i = v_j$. But since $\alpha_j(v, x_i) > 0$ by A2, and rejecting x_i moves the state back to v, player j accepts v_j immediately in real time (eventually in artificial time). This implies that i's expected value is $v_i = \pi_i(\sigma|h) = 1 - v_j$. Thus, $v_1 + v_2 = 1$. Conversely, any delay causes inefficiency. So if $v_1 + v_2 = 1$, at least one of the players i offers immediately, that is, $\lambda_i(v) = \infty$.

A.4 Approaching the Pareto Frontier: Proof of Theorem 3

Lemma 6 *Fix an AE σ. If $\bar{v}_i = \sup\{v_i \mid v \in \mathrm{A}_{IR}(\sigma)\}$, then $[(\bar{v}_1, 1-\bar{v}_1) + \mathbb{B}_\varepsilon] \cap \mathrm{A}_{IR}(\sigma) \neq \emptyset$ and $[(1 - \bar{v}_2, \bar{v}_2) + \mathbb{B}_\varepsilon] \cap \mathrm{A}_{IR}(\sigma) \neq \emptyset$ for all $\varepsilon > 0$, where $\mathbb{B}_\varepsilon$ denotes the ε ball in $\mathbb{R}^2$.*

WLOG, let $i = 2$. By contradiction, assume $[(1 - \bar{v}_2, \bar{v}_2) + \mathbb{B}_\varepsilon] \cap \mathrm{A}_{IR}(\sigma) = \emptyset$ for some $\epsilon > 0$. Consider the triangle Δ with vertices $(1 - \bar{v}_2, \bar{v}_2)$, $(1 - \bar{v}_2, 0)$ and $(1, 0)$. Note that v_2 is uniformly bounded away from $\bar{v}_2$ in the region $\Delta \setminus [(1 - \bar{v}_2, \bar{v}_2) + \mathbb{B}_\varepsilon]$. Therefore, by the definition of $\bar{v}_2$, there exists $v \in \mathrm{A}_{IR}(\sigma)$ with $v_1 < 1 - \bar{v}_2$ and $v_2 < \bar{v}_2$. Suppose that player 1 then offers $x \in (\bar{v}_2, 1 - v_1)$ (possibly off-path). If $\alpha_2(v, x) = 1$, then his final payoff is $1 - x > v_1$, and offering x immediately lifts his payoff, a contradiction. But if $\alpha_2(v, x) < 1$, then player 2 must expect a continuation value of at least $x > \bar{v}_2$. So there exists $w \in \mathrm{A}_{IR}(\sigma)$ with $w_2 > \bar{v}_2$, contrary to the definition of $\bar{v}_2$.

A.5 Exploding Proposal Rates in the Interior by Example

We now show that the proposal rate in an AE can explode at an aspiration vector v that is not on the Pareto frontier. We then provide a simple intuitive condition that rules it out.

Example (Exploding Proposal Rates): Assume that $r_1 = r_2 = 1$ and $v^0 = (3/8, 3/8)$. Consider the function $\varphi : (1/4, 1/2) \to (1/4, 1/2)$ given by $\varphi(x) = 3/4 - x$, with graph $\mathrm{G}(\varphi)$. We construct σ so that $\mathrm{A}(\sigma) = \mathrm{G}(\varphi) \cup \{(\tfrac{1}{4}, \tfrac{3}{4}), (\tfrac{1}{2}, \tfrac{1}{2}), (\tfrac{3}{4}, \tfrac{1}{4}), (\tfrac{1}{4}, \tfrac{1}{2}), (\tfrac{1}{2}, \tfrac{1}{4})\}$.

For all $v \in \mathrm{G}(\varphi)$, let $\rho_{ij}(v, x) = \varphi(x)$ and $\rho_{ii}(v, x) = x$ if $x \geq v_i$, and $\rho_i(v, x) = v$ if $x < v_i$. For any $v \in \mathrm{G}(\varphi)$, let $\mu_i(v)$ be a degenerate distribution at $\bar{x}_i(v)$, where

$$\bar{x}_i(v) = \frac{1}{2}\left[\frac{1}{2} + v_j\right], \quad \lambda_i(v) = \frac{4v_j}{1 - 2v_j}, \quad \alpha_j(v, x) = \begin{cases} 0 & \text{if } x < \bar{x}_i(v) \\ 2v_i - 1/2 & \text{if } x \geq \bar{x}_i(v). \end{cases}$$

When $v \in \{(1/4, 1/2), (1/2, 1/4)\}$, we continue as in the 'Battle of Sexes' example, to $(1/4, 3/4)$ or $(1/2, 1/2)$ from $(1/4, 1/2)$, and to $(1/2, 1/2)$ or $(3/4, 1/4)$ from $(1/2, 1/4)$.

As $v_i \downarrow 1/4$ and $v_j \uparrow 1/2$, we have $\lambda_i(v) \uparrow \infty$, $\kappa_i(v) \downarrow 0$, and $\alpha_j(v, \bar{x}_i(v)) \downarrow 0$. Thus, the AP $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ may generate a cluster point of offer times in finite time. In particular, for example, starting at $v^0 = (3/8, 3/8)$, consider an infinite history h where player 1 makes all the offers $\bar{x}_1(v^k)$ that player 2 always rejects, where $v^{k+1} = (\varphi(\bar{x}_1(v^k)), \bar{x}_1(v^k))$ for all $k \geq 0$. The set of such histories has probability $\Pi_{k=0}^{\infty}(1 - \alpha_2(v^k, \bar{x}_1(v^k)))\lambda_1(v^k)/[\lambda_1(v^k) + \lambda_2(v^k)] > 0$ (proof omitted). Along any such history h, offers arrive increasingly rapidly, becoming stingier and more likely to be rejected. The expected elapse time of the k-th offer by player 1 is $t_k = 1/\lambda_1(v^k) = 2^{-k}/[2(4 - 2^{-k})]$; therefore, the expected time of the cluster point generated by such a history is $\sum t_k \leq \sum 2^{-k}/6 = 1/3$. Let σ be the AE that coincides with $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$ for any finite history. For an infinite history $h \in \mathcal{H}_+$ ending with player i making a sequence of offers converging to $1/2$ in finite time (a cluster point), put $\sigma = \bar{\sigma}(v^i, \lambda, \mu, \alpha, \rho)$, where $v_i^i = 1/4$ and $v_j^i = 1/2$.

We noted in §3.1 that our AP does not fully describe strategies with cluster points. It is natural to ask how to rule out this possibility. In fact, the example works because surplus concessions κ_i and acceptance chances α_j both vanish as we approach $(v_i, v_j) = (1/2, 1/4)$. The next result shows that if we rule out this possibility, cluster points cannot obtain.

Lemma 7 *Let σ be an AE $\bar{\sigma}(v^0, \lambda, \pi, \alpha, \rho)$ such that for any closed subset A in the interior of* A*, there exists $\underline{\kappa} > 0$ such that $\bar{\kappa}_1(v), \bar{\kappa}_2(v) \geq \underline{\kappa} > 0$ for all $v \in A$. Then proposal rates $\lambda_1(v), \lambda_2(v)$ are bounded in the set A.*

Proof: The result follows from rewriting the delay IC equation (3) as

$$\lambda_j \bar{\kappa}_j = v_i r_i / (1 - v_1 - v_2). \tag{10}$$

A.6 Decreasing Aspiration Sets: Proof of Lemma 4

Assume that $v, v' \in \mathrm{A}(\sigma)$ with $v_j = v'_j$ and $v_i < v'_i$. Let $x_j \in \operatorname{supp}(\mu_j(v'))$ such that $x_j < 1 - v_j$ (guaranteed by $B2$). Then $v'_j = \alpha_i(v', x_j)(1 - x_j) + (1 - \alpha_i(v', x_j))\rho_{ij}(v', x_j)$, and since $\alpha_i(v', x_j) > 0$ (by A3) and $1 - x_j > v'_j$, we must have $\rho_{ij}(v', x_j) < v'_j$. Therefore, if player j makes the same offer x_j at v, he expects $\alpha_i(v, x_j)(1 - x_j) + (1 - \alpha_i(v, x_j))\rho_{ij}(v, x_j) > v'_j = v_j$ since $\alpha_i(v, x_j) > \alpha_i(v', x_j)$ by $B1$, and $\rho_{ij}(v, x_j) = \rho_{ij}(v', x_j)$ by $A2$. Hence, player j can raise his expected value v_j by offering x_j, a contradiction.

For each $w \in \mathrm{A}$ with $w_1 + w_2 < 1$, let $L(w)$ denote the right triangle with vertices w, $(w_1, 1 - w_1)$ and $(1 - w_2, w_2)$. Clearly if $L(w)$ and $L(w')$ contain $\mathrm{A}_{IR}(\sigma)$, then $\mathrm{A}_{IR}(\sigma) \subset L(w) \cap L(w')$. Also, there exists w'' such that $L(w) \cap L(w') = L(w'')$. Let L^* be the intersection of all triangles $L(w)$ containing $\mathrm{A}_{IR}(\sigma)$. Then $L^* \neq \emptyset$ since $v^0 \in L^*$, and clearly there exists w^* such that $L^* = L(w^*)$. Let $\underline{w} = w^*_1$ and $\bar{w} = 1 - w^*_2$. We now show that $\mathrm{A}_{IR}(\sigma)$ intersects every vertical and horizontal line through L^* once and only once. Suppose, eg., the vertical line at some level $v_1 \in (\underline{w}, \bar{w})$ does not intersect $\mathrm{A}_{IR}(\sigma)$: i.e. $\{v_1\} \times [0, 1] \cap \mathrm{A}_{IR}(\sigma) = \emptyset$. Then $[0, v_1] \times [0, 1 - v_1] \cap \mathrm{A}_{IR}(\sigma) = \emptyset$. For if w were in this intersection and player 2 offered $x_2 = v_1$ at w, then $\rho_1(w, x_2) \in \{v_1\} \times [0, 1] \cap \mathrm{A}_{IR}(\sigma)$ by B**??**. So $\mathrm{A}_{IR}(\sigma) \subset L(\underline{w}, 1 - v_1) \cup L(v_1, 1 - \bar{w})$. But for any $w \in L(\underline{w}, 1 - v_1)$, say, and any IR offer x_i at w, $\rho_j(w, x_i) \in L(\underline{w}, 1 - v_1)$. Thus, $\mathrm{A}_{IR}(\sigma) \subset L(\underline{w}, 1 - v_1)$ or $\mathrm{A}_{IR}(\sigma) \subset L(v_1, 1 - \bar{w})$, depending on which contains v^0, contrary to the definition of L^*.

If $\mathrm{A}_{IR}(\sigma)$ is not decreasing, there must exist two Pareto-ranked points $v, v' \in \mathrm{A}_{IR}(\sigma)$, say $v < v'$. Since v' is the only point in $\mathrm{A}_{IR}(\sigma)$ on the horizontal line through v', if player 1 offers $x_1 = v'_2$ at v, he gets $1 - x_1 \geq v'_1 > v_1$ if the offer is accepted and v'_1 otherwise. His expected value is thus strictly higher than v_1 after making that offer, a contradiction. So no two points in $\mathrm{A}_{IR}(\sigma)$ can be Pareto ranked, and $\mathrm{A}_{IR}(\sigma) \cup \{(\underline{w}, 1 - \underline{w}), (\bar{w}, 1 - \bar{w})\}$ is the graph of a decreasing function $\varphi : \bar{W} \rightarrow [0,1]$. Since $\mathrm{A}_{IR}(\sigma)$ intersects every vertical and horizontal line through L^*, φ is continuous, and $\varphi(\underline{w}) = 1 - \underline{w}$ and $\varphi(\bar{w}) = 1 - \bar{w}$.

A.7 Sensitivity Analysis: Proof of Theorem 5

Let σ be an AE $\bar{\sigma}(v^0, \lambda, \mu, \alpha, \rho)$. By Lemma 4, $\mathrm{A}_{IR}(\sigma)$ is the graph of a continuous decreasing function $\varphi : W \rightarrow [0,1]$, where $W = (\underline{w}, \bar{w})$. We will expand the set of states and view the bargaining process on the equilibrium path as a stochastic process, where agreement corresponds to absorption on the Pareto frontier. With each $w \in W$ we identify two states, w^A and w^B. The first represents the *absorbing* (or *agreement*) state $(w, 1-w)$ on the Pareto frontier, and the second the *transient* (or *bargaining*) state $(w, \varphi(w))$. Abusing notation, denote $\mu_i(w, \varphi(w))$ by $\mu_i(w)$.

For every $v \in \mathrm{A}$, let $\gamma(v) = \lambda_1(v)/[\lambda_1(v) + \lambda_2(v)]$ be the chance that in equilibrium player 1 makes the next offer at the aspiration vector v. Define $e(w) = 1 - w$, $w \in [0,1]$.

Below, we will exploit the fact that our bargaining process is really a mixture of the two artificial processes where only one of the players $i = 1, 2$ makes all the offers. Indeed, let $M_i^N(w, B)$ ($M_i^Y(w, B)$, resp.) be the chance that from an initial state $w \in W$, the next period's state is a transient (absorbing) state in $B \in \mathrm{B}(W)$, when player i makes all the offers (that is, when we set $\lambda_j \equiv 0$). Then,

$$M_1^N(w,B) = \int_{\varphi(B)} (1 - \alpha_2(w,x))\mu_1(dx|w), \;\; M_1^Y(w,B) = \int_{e(B)} \alpha_2(w,x)\mu_1(dx|w),$$
$$M_2^N(w,B) = \int_B (1 - \alpha_1(w,x))\mu_2(dx|w), \;\; M_2^Y(w,B) = \int_B \alpha_1(w,x)\mu_2(dx|w)$$

for all $w \in W$ and $B \in \mathrm{B}(W)$. The bargaining stochastic process is described by the following transition probability functions $M^N, M^Y : [0,1] \times \mathrm{B} \rightarrow [0,1]$:

$$\begin{aligned} M^N(w,B) &= \gamma(w)M_1^N(w,B) + (1-\gamma(w))M_2^N(w,B) \\ M^Y(w,B) &= \gamma(w)M_1^Y(w,B) + (1-\gamma(w))M_2^Y(w,B). \end{aligned}$$

Thus, $M^N(w,B)$ is the mixture of the chances $M_1^N(w,B)$ and $M_2^N(w,B)$, i.e. the chance that from an initial state $w \in W$, the next period's state is a transient state via B. This transition probability depends on both the random offers and the acceptance chances.

Let $\mathcal{K}$ be the set of stochastic kernels $K : W \times \mathrm{B}(W) \rightarrow [0,1]$, where for each $w \in W$,

$K(w,\cdot) \in \Delta(W)$, and for $B \in \mathrm{B}(W)$, $K(\cdot, B)$ is a measurable function. Each stochastic kernel K represents the transition probabilities from transient to absorbing states for some stochastic process. For each $w \in W$ and $B \in \mathrm{B}(W)$, $K(w, B)$ is the chance that when the process starts at the transient state w, it enters the set absorbing states in B. For any kernel $K \in \mathcal{K}$, let $\bar{K}(w) = \int_W xK(w, dx)$ be the expected absorbing state when the process starts at the bargaining state $w \in W$. For any two kernels $K_1, K_2 \in \mathcal{K}$, write $K_1 \preceq K_2$ if $\bar{K}_1(w) \leq \bar{K}_2(w)$ for all $w \in W$. The poset $(\mathcal{K}, \preceq)$ is a complete lattice.

We assume $\hat{r}_1/r_1 > \hat{r}_2/r_2$, so that $\hat{\gamma}(w) > \gamma(w)$ for all $w \in W$. Let Q (resp. $\hat{Q}$) be the kernel corresponding to the stochastic process generated by the AE σ (resp. $\hat{\sigma}$).

Define $\Psi : \mathcal{K} \to \mathcal{K}$ by

$$\Psi(K)(w, B) = \gamma(w)\Psi_1(K)(w, B) + (1 - \gamma(w))\Psi_2(K)(w, B)$$

for each $K \in \mathcal{K}$, where for $i = 1, 2$, we have

$$\Psi_i(K)(w, B) = \int_W K(x, B)M_i^N(w, dx) + M_i^Y(w, B)$$

for each $w \in W$ and $B \in \mathrm{B}(W)$. The stochastic kernel Q satisfies the standard equation $Q(w, B) = \Psi(Q)(w, B)$ for all $w \in W$ and $B \in \mathrm{B}$. Similarly, $\hat{Q}$ is a fixed point of the map $\hat{\Psi}$ defined using $\hat{\gamma}$ in place of γ.

Step 1 *If $Q \preceq \hat{\Psi}(Q)$, then $Q \preceq \hat{Q}$.*

Proof of Step 1: By Tarski's fixed point theorem, $\hat{Q} = \sup \{K \mid K \preceq \hat{\Psi}(K)\} \succeq Q$. $\square$

By Step 1, since $Q = \Psi(Q)$, it suffices to show that $\Psi(Q) \preceq \hat{\Psi}(Q)$. We have

$$\begin{aligned} \Psi(Q)(w, B) &= \gamma(w)\Psi_1(Q)(w, B) + (1 - \gamma(w))\Psi_2(Q)(w, B) \\ \hat{\Psi}(Q)(w, B) &= \hat{\gamma}(w)\Psi_1(Q)(w, B) + (1 - \hat{\gamma}(w))\Psi_2(Q)(w, B) \end{aligned}$$

for all $w \in W$ and $B \in \mathrm{B}(W)$. Since $\gamma(w) < \hat{\gamma}(w)$ for each $w \in W$, $\Psi(Q) \preceq \hat{\Psi}(Q)$ iff $\Psi_1(Q) \preceq \Psi_2(Q)$. We must prove that for any initial transient state, player 1 gets a larger expected share from the process where player 2 makes all offers than with reversed roles.

Step 2 *There exists a positive measure* ξ *on* $(W, \mathrm{B}(W))$ *and* $\beta \in (0,1)$ *such that*

$$\int_W M^N(w, B) d\xi(w) \leq \beta \xi(B) \quad \textit{for all } B \in \mathrm{B}(W),$$

and $\xi(B) \geq m(B)$ *for all* $B \in \mathrm{B}(W)$*, where* m *denotes Lebesgue measure.*

Proof of Step 2: Let $\mathcal{V}$ denote the set of signed measures of bounded variation on $(W, \mathrm{B}(W))$ with the total variation norm. The Banach space $\mathcal{V}$ is the dual of $L_\infty(W)$. Let $1 - \underline{\alpha} < \beta < 1$ (recall B**??**). Define $\Phi : \mathcal{V} \to \mathcal{V}$ by

$$\Phi(\xi)(B) = \frac{1}{\beta} \int_W M^N(x, B) d\xi(x) + m(B) \qquad \forall \xi \in \mathcal{V}, B \in \mathrm{B}(W).$$

Consider instead $\mathcal{V}$ with the weak-$*$ topology. It is easy to see that Φ is continuous when $\mathcal{V}$ has this topology. Indeed, suppose that $\{\xi_n\} \subset \mathcal{V}$ is such that $\xi_n \to \xi$ in the weak-$*$ topology. That is, $\langle f, \xi_n \rangle \to \langle f, \xi \rangle$ for all $f \in L_\infty(W)$, where, for example, $\langle f, \xi_n \rangle = \int_W f(x) d\xi_n(x)$. Fix $f \in L_\infty(W)$. Let $g(x) = [1/\beta] \int_W f(y) M^N(x, dy)$. Since $f \in L_\infty(W)$, we have $g \in L_\infty(W)$ as well. Therefore,

$$\begin{aligned} \langle f, \Phi(\xi_n) \rangle - \langle f, m \rangle &= \frac{1}{\beta} \int_W \int_W f(y) M^N(x, dy) d\xi_n(x) \\ &= \int_W g(x) d\xi_n(x) = \langle g, \xi_n \rangle \to \langle g, \xi \rangle = \langle f, \Phi(\xi) \rangle - \langle f, m \rangle. \end{aligned}$$

Let C be the set of all positive measures ξ on $(W, \mathrm{B}(W))$ such that $\xi(W) \leq \beta m(W)/[\beta - (1 - \underline{\alpha})]$. Let $\xi \in C$. Then $\Phi(\xi) \in C$ because

$$\begin{aligned} \Phi(\xi)(W) &= \frac{1}{\beta} \int_W M^N(x, W) d\xi(x) + m(W) \leq (1 - \underline{\alpha}) \xi(W)/\beta + m(W) \\ &\leq \frac{1 - \underline{\alpha}}{\beta - (1 - \underline{\alpha})} m(W) + m(W) = \beta m(W)/[\beta - (1 - \underline{\alpha})]. \end{aligned}$$

Thus, $\Phi(C) \subset C$. As C is a convex, bounded and closed subset of $\mathcal{V}$ (with the total variation topology), it is weak-$*$ compact by Alaoglu's Theorem. By Schauder's Fixed Point Theorem, $\Phi : C \to C$ has a fixed point ξ. Finally, $\xi = \Phi(\xi)$ implies that for all $B \in \mathrm{B}(W)$, $\xi(B) \geq m(B)$ and

$$\textstyle\int_W M^N(x, B) d\xi(x) = \beta[\xi(B) - m(B)] \leq \beta \xi(B). \qquad \square$$

Let L be the space of functions $K : W \times \mathrm{B} \to \mathbb{R}$ such that for each $w \in W$, $K(w, \cdot)$ is a signed Borel measure on W, and for each $B \in \mathrm{B}(W)$, $K(\cdot, B)$ is a (Borel) measurable function. Thus, $\mathcal{K}$ is a subset of the positive cone in L. Endow L with the norm

$$\|K\| = \int_W \int_W |K(w, dx)| d\xi(w) = \int_W |K(w, \cdot)|(W) d\xi(w),$$

where $|K(w, \cdot)|$ is the positive measure $K_+(w, \cdot) + K_-(w, \cdot)$. Then, L is a Banach space.

Step 3 *$\mathcal{K}$ is a closed subset of* L *and $\Psi : \mathrm{L} \to \mathrm{L}$ is a contraction.*

Proof of Step 3: Pick $K, K' \in \mathrm{L}$ and let $D(w, B) = |K(w, \cdot) - K'(w, \cdot)|(B)$ for all $w \in W$ and $B \in \mathrm{B}(W)$. For each $B \in \mathrm{B}(W)$, we have that

$$\Psi(K)(w, B) - \Psi(K')(w, B) = \int_W [K(x, B) - K'(x, B)] M^N(w, dx).$$

Hence

$$\begin{aligned} \|\Psi(K) - \Psi(K')\| &\leq \int_W \int_W D(x, W) M^N(w, dx) d\xi(w) \\ &\leq \beta \int_W D(w, W) d\xi(w) = \beta \|K - K'\|. \end{aligned} \qquad \square$$

We projected the aspiration set $\mathrm{A}_{IR}(\sigma)$ into the horizontal axis to construct the one-dimensional bargaining state space W. Alternatively, since φ is strictly decreasing, we could have projected $\mathrm{A}_{IR}(\sigma)$ into the vertical axis to produce a "dual" state space $W^* = \varphi(W) = (1 - \bar{w}, 1 - \underline{w})$. For each kernel $K \in \mathcal{K}$ there corresponds a dual kernel K^*, defined by $K^*(w, B) = K(\varphi^{-1}(w), e(B))$ for each $w \in W^*$ and $B \subset \mathrm{B}(W)$. When player 2 has an initial aspiration value w, player 1 has the initial aspiration value $\varphi^{-1}(w)$, and player 2's final share is in B iff player 1's share is in $e(B)$. Thus, $K^*(w, B)$ represents the probability that when player 2's expected value is w, his share in the final split is in the set B.

Step 4 *Let $\hat{\mathcal{K}}$ be the set of kernels $K \in \mathcal{K}$ such that (i) $\bar{K}(w) \geq w$ for all $w \in W$; (ii) $\bar{K}^*(w) \geq w$ for all $w \in W^*$. Then the fixed point Q of Ψ belongs to $\hat{\mathcal{K}}$.*

Proof of Step 4: $\hat{\mathcal{K}}$ is a closed subset of L. Hence, by Step 3, it is enough to show that $\Psi(K) \in \hat{\mathcal{K}}$ for all $K \in \hat{\mathcal{K}}$.

Note that $\varphi^{-1}(w)$ is the new bargaining state when player 1 makes the admissible offer w and player 2 rejects it. Fix $K \in \hat{\mathcal{K}}$ and denote $J = \Psi(K)$. We first show that J satisfies constraint (i) above. We have $J(w,\cdot) = \gamma(w)J_1(K)(w,\cdot) + (1-\gamma(w))J_2(K)(w,\cdot)$, where $J_i = \Psi_i(K)$. Therefore,

$$\begin{aligned}
\bar{J}(w) &= \gamma(w)\int_{\varphi(W)}[(1-\alpha_2(w,x))\bar{K}(\varphi^{-1}(x)) + \alpha_2(w,x)(1-x)]\mu_1(w)(dx) \\
&\quad + (1-\gamma(w))\int_W[(1-\alpha_1(w,x))\bar{K}(x) + \alpha_1(w,x)x]\mu_2(w)(dx) \\
&\geq \gamma(w)\int_{\varphi(W)}[(1-\alpha_2(w,x))\varphi^{-1}(x) + \alpha_2(w,x)(1-x)]\mu_1(w)(dx) \\
&\quad + (1-\gamma(w))\int_W[(1-\alpha_1(w,x))x + \alpha_1(w,x)x]\mu_2(w)(dx) \\
&\geq \gamma(w)w + (1-\gamma(w))\bar{x}_2(w) \geq w,
\end{aligned}$$

because $w = (1-\alpha_2(w,x))\varphi^{-1}(x) + \alpha_2(w,x)(1-x)$ for each $x \in [\varphi(w), 1]$, by (4) in Theorem 2, and because $\text{supp}(\mu_2(w)) \subset [w,1]$ (so $\bar{x}_2(w) \geq w$) by Theorem 2. A symmetric argument shows that J also satisfies (ii). Therefore, $Q \in \hat{\mathcal{K}}$. $\square$

Step 5 *Properties* (i) *and* (ii) *imply* $\Psi_1(K) \preceq \Psi_2(K)$ *for all* $K \in \hat{\mathcal{K}}$.

Proof of Step 5: Pick any $K \in \hat{\mathcal{K}}$ and as before denote $J_i = \Psi_i(K)$, $i = 1, 2$. Since the sum of the players' expected final shares cannot exceed 1, player 2's expected share starting at $w \in \bar{W}$ cannot exceed $1 - \bar{K}(w)$. By (ii), this expected share is at least $\varphi(w)$ (since the aspiration value of w for player 1 corresponds to the aspiration value $\varphi(w)$ for player 2). So $1 - \bar{K}(w) \geq \varphi(w)$ or $\bar{K}(w) \leq 1 - \varphi(w)$. In particular, $\bar{K}(\varphi^{-1}(w)) \leq 1 - w$. Hence

$$\begin{aligned}
\bar{J}_1(w) &= \int_{\varphi(W)}[(1-\alpha_2(w,x))\bar{K}(\varphi^{-1}(x)) + \alpha_2(w,x)(1-x)]\mu_1(w)(dx) \\
&\leq \int_{\varphi(W)}[(1-\alpha_2(w,x))(1-x) + \alpha_2(w,x)(1-x)]\mu_1(w)(dx) = 1 - \bar{x}_1(w)) \\
\bar{J}_2(w) &= \int_W[(1-\alpha_1(w,x))\bar{K}(x) + \alpha_1(w,x)x]\mu_2(w)(dx) \\
&\geq \int_W[(1-\alpha_1(w,x))x + \alpha_1(w,x)x]\mu_2(w)(dx) = \bar{x}_2(w)).
\end{aligned}$$

By B3, $\bar{x}_2(w) \geq 1 - \bar{x}_1(w)$, and thus $\Psi_1(K) = J_1 \preceq J_2 = \Psi_2(K)$. $\square$

Since $Q \in \hat{\mathcal{K}}$, we have $\Psi_1(Q) \preceq \Psi_2(Q)$, and therefore $Q \preceq \hat{Q}$, as required. $\square$

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