

## Monotone Comparative Statics

This half term course studies modern lattice-theoretic methods of supermodularity and comparative statics. We then explore stochastic orders, and extend the monotone comparative statics to uncertainty. For this, we explore total positivity, including log-supermodularity. Finally, we contrast the foundations and recent work on value and demand for information, and how to rank informative signals.

**Pre-requisites.** First year PhDs can take the course. If desired, it can substitute for Econ 609 in the advanced math methods requirement; this substitution makes sense for non-macroeconomists. The main advantage conferred by second or higher year status is intellectual maturity, and an ability to leap tall buildings in a single bound.

**Grading.** For a grade, the major course credit hurdle is a final exam, worth 60% of the grade, on the last day of class, **Thursday, February 28**. The rest of the grade comes from two problem sets, due Mondays: **February 4, February 25**. Prospective exam/prelim takers should seriously solve them, since *those who don't solve problems simply don't learn Theory!* My (usually joint) papers often start out as assignments or exams.

**Texts.** Topkis is useful to own, even though I largely summarize its content; I assume that those who want it got it for 617. If not, order it online, and it should arrive in a week. Check out [www.pricescan.com](http://www.pricescan.com) for lowest prices. Any needed papers will be placed in Foster as they are needed. If you want them earlier, my papers are on my web page, at [www.umich.edu/~lones/papers.html](http://www.umich.edu/~lones/papers.html). Books below should soon be on reserve.

- Morris DeGroot, Optimal Statistical Decisions. 1970.
- DJ: Sudhakar Dharmadhikari and Kumar Joag-dev, Unimodality, Convexity, and Applications. Academic Press, 1988.
- HR: Jack Hirshleifer and John Riley, The Analytics of Uncertainty and Information, Cambridge University Press, 1992.
- Samuel Karlin. Total Positivity, vol. 1, Stanford University Press. 1968.
- MO: Marshall and Olkin, Inequalities: Theory of Majorization and its Applications, Academic Press, 1979.
- SS: Moshe Shaked and George Shanthikumar, Stochastic Orders and their Applications, Academic Press, 1994.
- ★ Donald Topkis, Supermodularity and Complementarity, Princeton University Press, Frontiers of Economic Research Series, 1998.

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- = very early reference on subject material
  - = good modern background reading
  - ★ = required reading, possibly in Topkis
  - ⊙ = required reading, and ideal fireside reading to boot.
  - ◇ = related good reading, but I will skip it due to time pressures

I. Comparative Statics, Risk, and Uncertainty ( $\approx 8$  lectures)

A. Comparative Statics under Certainty ( $\approx 2$  lectures)

□ IDEAS: SUPERMODULARITY AND QUASI-SUPERMODULARITY

- ★ Topkis, chapter 2: “Lattices, Supermodular Functions, and Related Topics”
- ★ Paul Milgrom and Chris Shannon (1994), “Monotone Comparative Statics,” *Econometrica*, **57**: 157–180.

B. Stochastic Orders ( $\approx 2$  lectures)

□ IDEAS: DUALITY AND THE METHOD OF CONES

- Erich Lehmann (1955), “Ordered Families of Distributions,” *Annals of Mathematical Statistics* **26**: 399–419.
- Hardy, Littlewood, and Polya, *Inequalities*, 2nd edition, 1952, §2.20.
- David Schmeidler (1979), “A Bibliographical Note on a Theorem of Hardy, Littlewood, and Polya”, *Journal of Economic Theory* **20** (1): 125–28.
- ★ Michael Rothschild and Joseph Stiglitz (1970), “Increasing Risk I: A Definition,” *Journal of Economic Theory*, **2**: 225–243.
- ★ Topkis, §3.9.1, 3.10.1; Karlin, §6.2; SS §1.A, 2.A, 3.A, 4.B, 5.A
- S. L. Brumelle and R.G. Vickson (1975), “A Unified Approach to Stochastic Dominance”, in *Stochastic Optimization Models in Finance*, New York, Academic Press.
- ★ Susan Athey (2000), “Characterizing Properties of Stochastic Objective Functions,” MIT working paper.

C. Comparative Statics Under Uncertainty ( $\approx 3$  lectures)

□ KEY IDEAS: LOG-SUPERMODULARITY; LOG-CONCAVITY; SINGLE-CROSSING AND VARIATION-DIMINISHING PROPERTIES

- ★ Karlin, §1.1–3, 3.5, 4.1, 6.3; MO, §18.A–B; DJ, chapter 2.
- ◇ Karlin and Rinott (1980), “Classes of Orderings of Measures and Related Correlation Inequalities. I. Multivariate Totally Positive Distributions,” *Journal of Multivariate Analysis* **10** (4): 467–498.
- ★ Susan Athey (2000), “Monotone Comparative Statics Under Uncertainty”, forthcoming in the *Quarterly Journal of Economics*.
- Peter Diamond and Joseph Stiglitz (1974), “Increases in Risk and in Risk Aversion”, *Journal of Economic Theory* **8**: 337–60.
- ⊙ “Single-Crossing Property for Gambles” (Lemma 6 in Shimer-Smith (2000))
- ★ Ken Burdett (1996), “Truncated Means and Variances”, *Economics Letters* **52**: 263–67. [cf. Karlin (1968), §6.11.H]

D. Comparative Dynamics ( $\approx 1$  lecture)

□ IDEA: TARSKI FIXED POINT THEOREM AS A COMPARATIVE STATIC TOOL

- Lones Smith and Ennio Stacchetti, “Comparative Statics of Markov Processes” (2001), private notes, plus the appendix of “Aspirational Bargaining” (2002)

II. Information ( $\approx 6$  lectures)

A. Ex Post Informative Signals Comparison ( $\approx 1$  lecture)

□ IDEA: LIKELIHOOD RATIO ORDERING

- Samuel Karlin and Herman Rubin (1956), “Distributions Possessing a Monotone Likelihood Ratio”, *Journal of the American Statistical Association*, **51**: 637–643.
- Ward Whitt (1979), “A Note on the Influence of the Sample on the Posterior Distribution,” *Journal of the American Statistical Association*, **74** (366): 424–426.
- ★ Ward Whitt (1980), “Uniform Conditional Stochastic Order”, *Journal of Applied Probability*, **17**: 112–123.
- ★ Paul Milgrom (1981), “Good News and Bad News: Representation Theorems and Applications,” *Bell Journal of Economics*, **12**: 380–391.
- SS, §1.C,4.E

B. Application to Financial Trade Timing ( $\approx 1$  lecture)

□ IDEA: THE CONDITIONAL MARTINGALE

- ⊙ Lones Smith (2000), “Private Information and Trade Timing,” *The American Economic Review*, **90** (4): 1012–1017.

C. Ex ante Value of, and Demand for, Information ( $\approx 3$  lectures)

□ IDEAS: BLACKWELL’S THEOREM; MONOTONE LOCATION EXPERIMENTS; LARGE DEVIATION THEORY

- HR, §5.1–2: “Information and Informational Decisions.”
- ★ David Blackwell (1953), “Equivalent Comparison of Experiments,” *Annals of Mathematics and Statistics*, **24**: 265–272.
- Roy Radner and Joseph Stiglitz (1984), “A Nonconcavity in the Value of Information” in *Bayesian Models in Economic Theory*, Elsevier Science Publishers, New York, Marcel Boyer And Richard Kihlstrom, eds., pp. 33–52.
- ★ Edward Schlee and Hector Chade (2001), “Another Look at the Radner-Stiglitz Nonconcavity in the Value of Information,” forthcoming, *Journal of Economic Theory*.
- Erich Lehmann (1988), “Comparing Location Experiments”, *Annals of Statistics*, **16**: 521-33.
- ★ Susan Athey and Jonathan Levin (2000), “The Value of Information in Monotone Decision Problems”, MIT Working Paper.
- ★ Giuseppe Moscarini and Lones Smith (2000), “The Demand for Normal Information”, private notes (possibly locally handed out, maybe not).
- ★ Giuseppe Moscarini and Lones Smith (2001), “The Law of Large Demand for Information,” revised for *Econometrica*.

D. Fixed Points and Existence of Equilibrium in Games ( $\approx 1$  lecture)

- Athey, “Equilibrium Existence in Games of Incomplete Information”