Lones Smith due Mon., Mar. 26, 2001

Econ 618: Assignment 3

1. Suppose there is a fair prior on the two states H and L. Let there be two actions A and B. Assume the standard symmetric payoffs ($u_B^H = u_A^L = 1, u_A^H = u_B^L = -1$), so that one takes action B, iff the posterior belief (not private belief!) on H lies above 1/2. Assume the standard uniform full support signals, with $F^H(x) = x^2$ and $F^L(x) = 2x - x^2$. In the standard informational herding game, what is the chance of a correct herd from the beginning (everyone does the right thing)? Compute the chance unconditional on knowledge of the state (i.e. using the fair prior).

   Hint: A closed form of the resulting infinite product you will find involves a famous formula for π that you do not need to know. So just leave it in the nicest possible infinite product form.

2. Consider a model with a continuum $(0, 1)$ of individuals, all Russian, and all sitting in on 618 for the second time. Assume that everyone initially has priors 50-50 over states H and L. Now let them all observe a random signal $s \in [0, 1]$. Let individuals know that the density of signals is $m_H(s) = 1$ in state H and $m_L(s) = -\log s$ in state L. Assume that the state is H.

   (a) After the signals have been realized, let the cross-sectional distributions of posteriors in the two states be F^L, F^H. Explicitly solve for F^H.

   (b) Assume that after observing the signal, individuals then have to guess the state. If they guess right, their payoff is 1; if they guess wrong, their payoff is -1. *Exactly* what is a typical Russian willing to pay for his signal (*before having seen it*).
   Hint: Part (a) is not needed to do this calculation.

   For (c) and (d), let everyone observe a further informative signal $\tilde{\tau}$ about the state.

   (c) Prove or disprove that this shifts F^H right in the sense of first order stochastic dominance.

   (d) Prove or disprove that this shifts $F^H + F^L$ left in the sense of second order stochastic dominance.

3. Testimony has ended, and jurors are cloistered in a conference room, deciding whether to convict or acquit a defendant. As a result of the trial testimony and arguments, each of the n jurors $j = 1, 2, \ldots, n$ has received a private signal s_j with range $\{\gamma, \iota\}$, where signal outcome γ favours the state $G = \{\text{accused is guilty}\}$, and ι favours the state $N = \{\text{accused is not guilty (i.e. is innocent)}\}$. A priori, the states are equilikely. It is common knowledge that signals are conditionally iid. In state G, $s_j = \gamma$ with chance 2/3, and $s_j = \iota$ with chance 1/3. In state N, $s_j = \iota$ with chance θ, and $s_j = \gamma$ with chance $1 - \theta$. Assume $\theta > 1/2$.

Assume the jurors' decision mechanism employs a one-shot game: In the conference room, each privately votes 'convict' or 'acquit' using their private signal alone. Majority rule prevails (and n is odd).

(a) Deduce whether the juror's voting strategies are *sincere* — i.e. each votes to convict iff he receives the signal outcome $s = \gamma$. If your answer depends on θ, give your answer parametrically. Explain intuitively what is going on.

(b) Assume that there is no jury, but that an unbiased judge (i.e. with a fair prior) alone decides. Assume that the payoff to making the right decision is one, while the payoff of the wrong decision is zero. What is the above signal s worth to him?