Lones Smith due Mon., Feb. 26, 2001

Econ 618: Assignment 2

Note 1: Brevity is appreciated, but be sure not to wave your hands in a proof.
Note 2: Unlike assignment 1, the only non rote question is #5 (a challenge).

1. Prove the following strengthening of Lehmann's Composition Theorem (Karlin 3.5.1): Assume that the functions $v(x,y,z) > 0$ and $w(x,y,z) > 0$ are each pairwise TP_2, fixing each of the third variables, fixing the third. Let μ be a positive measure. Then $h(x,z) = \int v(x,y,z)w(x,y,z)d\mu(y)$ is TP_2. What strict positivity assumptions are needed? [10 pts]

   Hint: Modify the proof of Lehmann's Composition Theorem that I handed out in class.

2. Suppose that $\int v(x,y)d\mu(y) = 1$ for all x. If $v(x,y)$ is TP_2, and $w(y_1, y_2, \ldots, y_n)$ is increasing coordinatewise, then [10 pts]

$$u(x) \equiv \int v(x,y_1)v(x,y_2)\cdots v(x,y_n)w(y_1,y_2,\ldots,y_n)d\mu(y_1)\cdots d\mu(y_n)$$

   is increasing in x (assuming the integral exists).

   Hint: Use an induction argument on n to extend our result from class for $n = 1$.

3. Assume a given interior prior belief over two states of the world. Show that Bayes-updating using a Blackwell more informative signal results in a posterior that is more spread in the sense of second order stochastic dominance. [10 pts]

4. This is based on my "Law of Large Demand for Information". In a three action, three state world with each action optimal in some state, express the gap between the full information payoff and the expected payoff of a partially informative signal as a simple weighted average of the different error chances. [10 pts]

5. [Challenge][1] Assume that f is a an increasing and twice differentiable function on $(0,1)$. Find robust conditions on f (like log-concavity) such that for all $0 < a < b < 1$, we have

$$\int_a^b \frac{f(x)f''(x) - f'(x)^2}{f(x)}dx \leq \frac{(\int_a^b f''(x)dx)(\int_a^b f(x)dx) - (\int_a^b f'(x)dx)^2}{(b-a)f(a)f(b)}$$

   Hint: Try ideas like log-concavity and results like Prekopa's Theorem, or the result of Ahlwede-Daykin (proved cleanly by Karlin-Rinott) that I cited without proof in class, and that Athey (2000) also uses. Experiment with functional forms.

[1] A challenge question means (*i*) it might be medium hard or very hard, but is likely not trivial; (*ii*) it is possibly a very useful result to me personally, for which I do not have the solution (but have not really focused on, so it might be very doable); and thus (*iii*) my posterior on your unobserved caliber would rise substantially, or possibly skyrocket (depending on how nifty the argument is) if you could get it. Even a delayed solution (eg. some time before the end of term) would have a similar effect.

Practice Questions from Past Years (not due!)

1. [**final exam, 1998**] Assume two states, $i = 1, 2$. Assume that a state-dependent signal θ can equal either outcome A, B, C with respective chances $a_i, b_i, c_i > 0$ in state i. Assume a decision-maker who makes a one-shot decision.

   (a) Characterize all signals with outcomes A, B, C that are not worth as much to all decision-makers as θ.

   (b) Characterize all signals with outcomes A, B, C that lead all decision-makers to place more weight on state $i = 2$ than θ.

2. [**prelim exam, 1999**] Consider a two-state world. Provide two binary outcome signals σ and τ such that some Bayesian decision makers (each parameterized by a prior belief vector, and state contingent payoffs) will strictly prefer σ to τ, and some will strictly prefer τ to σ.