Winter 2001 Lones Smith

Advanced Theory 618 — Final Exam

This take-home exam consists of 100 points (plus 20 bonus) due Monday 12pm in my office. To put you at ease, you start with 30 free points. Get lots of sleep! Take it easy. Try to impress both me, *and yourself.* You may only get help on this exam from me, but you can read your notes and articles. I also would rather offer a small hint that would lead you to discover the solution than that you just give up. Only five (5) pages will be graded, so don't hand in more. Cite known theorems from class or the class packet whenever helpful to save space and time (eg. in at least one case, you can just cite a result). Be rigorous when you can — i.e. if there is no standard theorem to cite.

If you have questions, see, email me, or phone me at home (994-9697) Sunday night. At the worst, my elder son could offer some help. ;-0

It was a pleasure having you in my class, and I hope you have acquired insights, and modelling tools that will serve you well.

"What is a magician but a practising theorist?" – Obi-Wan Kenobi

1. Suppose that one wishes to maximize consumer surplus $u(x)-p\cdot x$ for bundles $x \in \Re^n$ given prices $p \in \Re^n_+$. All that is given is that u is monotonic increasing in x. Assume that some prices rise. Can you say what happens to demand if $n > 1$? What if $n = 1$? [10]

2. Two jars have biased pennies. Jar A has 20 pennies that come up heads 2/3 of the time, and 10 that come up heads 1/3 of the time. Jar B is the reverse. It has 10 pennies that come up heads 2/3 of the time, and 20 just 1/3 of the time. [10]

   A coin will be drawn with equal chance from either jar, and if you guess the outcome of the coin flip, you win \$90. Assume you are risk neutral. What are you willing to pay to find out from which jar the coin was drawn?

3. [Challenge/Bonus] We learned in the course how the minimum of the Hellinger transform $\mathcal{H}(t)$ (across $t \in (0,1)$) of a signal (i.e. the MGF of the signal log-LR) is the right measure of the value of a signal as part of a large sample of iid replicas. But, I ask, is there a sense in which this is also the right measure for the value of the signal by itself? *Namely,* answer the following: does there exist a simple utility function $u(p,t)$ for beliefs p and actions t, such that maximizing $-\mathcal{H}(t)$ corresponds to maximizing $u(p,t)$ in t, and the utility maximum equals the maximum of $-\mathcal{H}(t)$.[1] [20]

4. Consider a herding model where private signal outcomes σ are distributed uniformly on $[0,1]$ in state H, and with density $g(\sigma) = 6\sigma(1-\sigma)$ on $[0,1]$ in state L.
   (a) What are the cdfs of private beliefs in states L and H? [5]
   (b) Describe the long-run behavior of the model, vis-à-vis completeness of learning. [5]
   Hint: It is possible to answer this part, without knowing (a).

[1] The answer may be no; I have not checked this, but it seems teasingly intriguing, and possibly very simple. Also, you may wish to adjust my statement of the question, if you find a better one.

5. Assume that wages are uniformly distributed on $(0, M)$, for some $M < 1$. Assume that your prior on M is uniform on $(0, 1)$.

   (a) If you see a given wage w', what is the posterior on M? [2]

   (b) Assume that you can either accept the wage, or stay unemployed, and have a opportunity to search next period (you will see only one wage each period you are searching). Let searches have no explicit costs, but assume that time is costly, as you are impatient with discount factor $\delta \in (0, 1)$. Set up the dynamic programming problem. [2]

   (c) *Precisely* (i.e. not just qualitatively) describe your optimal dynamic behavior. Among the questions you should be able to answer here are: Would you ever be willing to accept a wage, but later on, prefer to turn it down, because you are more optimistic? [16]

6. Assume a continuum of risk-neutral infinite lived agents who discount the future at rate $r > 0$. Agents identified by type $i \in \{1, 2\}$; ℓ_i is the measure of type i agents; $m_{ij}(t)$ is the measure of i's matched with j's at time t; $u_i(t) \equiv \ell_i - \sum_j m_{ij}(t)$ is measure of unmatched type i agents.

   An unmatched agent meets someone at rate ρ ('linear search technology'), and so enjoys a conditional probability $\gamma_j(t) \equiv u_j(t)/(u_1(t) + u_2(t))$ of meeting type j. If (i, j) match, they produce $f_{ij} \equiv f_{ji}$ and may not search. The match ends exogenously at rate $\delta > 0$. [20]

   I have kept time arguments since the point of this question is an *out-of-steady-state* analysis. So let me hold your hand for a little bit, and bore you with stuff you could have deduced. The time-t surplus $S_{ij}(t) = S_{ji}(t)$ from an ij match equals

$$S_{ij}(t) = \max_{T \in [t, \infty]} \int_t^T e^{-(r+\delta)(s-t)} \frac{f_{ij} - w_i(s) - w_j(s)}{2} ds$$

   where the value of an unmatched worker is $w_i(t) = \rho \sum_j \gamma_j(t) S_{ij}(t)$, i.e. it is the value of the option on future match surpluses. Finally, the evolution of state variables follows from matching behavior: If $S_{ij}(t) > 0$, then $\dot{m}_{ij}(t) = \rho u_i(t) \gamma_j(t) - \delta m_{ij}(t)$; if $S_{ij}(t) = 0$, then $0 \leq m_{ij}(t) \leq m_{ij}(t-)$.

   Question: Prove that moment-by-moment assortative matching occurs provided f is SPM, i.e. if ever types 1 and 2 are willing to matched should they meet (or willing to stay matched, if matched), then so must 1 and 1, and also 2 and 2.

   Hint: You should be able to trivially deduce this assuming a steady-state. Now assume that due to dynamics, some matches will be dissolving at a known future time, possibly different for different matches. Now proceed in a simple insightful fashion. You need not get your hands really dirty, if you see the trick.