Economics Game Theory 617: Assignment 2

Lones Smith due Monday, October 21 by email

Note: All solutions must be emailed to me (lones@umich.edu). The subject header must be included as `617 asmt`. Assignments typeset (in LaTeX, Word, etc.) will get a 10% bonus. Please do not email me (1) more than 4 pages (except for bonus material), and (2) files of size more than 200K. Please be rigorous. **Should you work in a group (not en/dis - couraged), please write up the solutions solo, and list your group members.**

Score denominators may be adjusted if I misjudge the difficulty of a question.

1. You are on Final Jeopardy in the championships, and money scores are $a > b > c > 0$. A question category is posed for which all candidates are equally skilled, and will get it with chance $p \in (0,1)$, independently across players. You may bet up to your current money score (in increments of \$1, on the show, but here any nonnegative real bet is permitted, for simplicity). If you bet x, your score jumps by x if you correctly answer the question; it falls by x, if you fail. The winner gets the \$1M prize (so the winning dollar score is irrelevant); with a tie, the prize is equally split. Assume the game is not a 'runaway' ($a < 2b$). [10]

   (a) Find with proof all rationalizable bets, *for each such fixed* $p \in (0,1)$.

   (b) (Warm-up) Deduce the significance of the threshold $b = 2/3a$ (that is widely known in Jeopardy circles).

   (c) Fixing the two other players' scores in the indicated range, *for what range of* a, b, c is one's expected payoff a strictly increasing function of one's score?

   Hint: This is a constant sum game, and so has unique Nash payoffs for given (a, b, c). It may help if you graph them. WLOG, you can scale $a = 1$.

   Note: If the problem is too hard, assume just two players with scores $a > b$.

   (d) While payoffs are uniquely defined, strategies may not be. Inasmuch as they are, supply any betting advice.

2. In a two player game, Nature moves first, choosing player 1's type to be rational (chance .7) or compulsive (chance .3). Player 1 observes his type, but player 2 only knows the probabilities (assumed common knowledge). Then player 2 moves, choosing 'stop' (payoff $(0,0)$) or 'go', giving the move to player 1. At that point, the compulsive player 1 has no choice except 'go', while the rational player 1 can 'go' or 'stop': 'Stop' yields payoff $(2,-1)$, and 'go' gives the move back to player 2. Player 2 can 'stop', or go: Stop yields payoff $(1,1)$, and 'go' gives the move back to player 1. Again, the compulsive player 1 can only 'go' (payoff $(2,2)$), while the rational player 1 can 'go' (payoff $(2,2)$) or 'stop' (payoff $(3,0)$). [5]

   (a) Represent this game in normal form. Find with proof *all* Bayes Nash equilibria.

   (b) Represent this game in extensive form. Find with proof *all* sequential equilibria.

3. (**Effort in Dynamic Games**) Consider a two-player game of tennis, with the match tied in the last game of the last set of Wimbleton. You need to be two points ahead to win the match (prize $\pi > 0$). Thus, the state variable is either $-2, -1, 0, 1, 2$, and the game ends when we hit -2 or 2. Each time you score a point, you advance one in this state space. Effort levels α, β are in $[0, \infty)$. Effort level a 'costs' a (so that this is subtracted from payoffs). If you and I exert effort levels $\alpha \geq 0$ and $\beta \geq 0$, respectively, with $\alpha + \beta > 0$, then you score the next point with chance $\alpha/(\alpha + \beta)$ and I with chance $\beta/(\alpha + \beta)$. We each try to maximize the undiscounted expected prize money less the sum of effort costs. [10]

(a) Solve for the *subgame perfect Nash equilibrium* strategies (i.e. describe behaviour in each state) and values, where effort levels are functions of the state — a Markov assumption, that is without loss of generality. In particular, determine the shape of the plot (both the monotonicity and convexity/concavity properties) of effort against the state. When people do try hardest?

Hint: Write down the Bellman equation for a player.

(b) Bonus: Can you generalize the general result on effort levels to a game with

- $2n + 1$ and not just 5 states, and
- a winning chance $Q(\alpha, \beta)$ obeying $Q_\alpha, Q_\beta > 0$, $Q_{\alpha\alpha}, Q_{\beta\beta} < 0$ (and I am not sure of the assumption on $Q_{\alpha\beta}$).