

Econ 610: Stochastic Dynamic Optimization in Economics

Overview. The goal of this course is to help you write exciting research papers that naturally involve both dynamics and uncertainty. While it is technical, it is *not* a micro theory course, since the domain of problems has little intersection with micro theory. The course consists of two parts, one-third discrete and two-thirds continuous time. The analysis will be generally overly loose, but I will seriously develop the first stochastic integral. If you use this stuff, you really will likely have to more carefully revisit these topics.

I really want the math to be extremely useful, and to affect the types of problems that many of you are willing to attack in your thesis. I show how to exploit one big idea throughout discrete and continuous time stochastic dynamic optimization, which is simply:

$$\text{Return} = \text{Dividend} + \text{Expected Capital Gain}$$

Texts. This is a course that exists nowhere else in the universe (at the level I am pitching it), and consequently has no ideal text. So please do not blame me for this! The most relevant texts that I have chosen are

- Sheldon Ross, “Introduction to Stochastic Dynamic Programming” (first two weeks)
- Avinash Dixit, “The Art of Smooth Pasting” (last four weeks)
- Oksendal [O], “Stochastic Differential Equations” (also useful for the last four weeks)

Last year, I required “Introduction to Stochastic Calculus with Applications” by Fima Klebener [K], which was more readable but skipped optimization. Dixit and Pindyck’s “Investment Under Uncertainty” is also a possible text, but it has a distracting agenda having nothing to do with the methods, and is therefore cluttered with silly minutiae. Still, you may like it.

Grading. Three **quizzes March 15, March 29, April 12** will comprise 36% of your term grade. One incredibly useful computational assignment showing how to simulate and find the solutions of stochastic dynamic programming problems that are too hard to solve (**due March 22**) will be worth 9%. The final exam will contribute 55%. As in all courses I have ever taught, the final grade of 610 entails a *term test forgiveness policy*: Even if you screwed up the early quizzes, your incentives are still to optimize on the rest, because

$$\text{grade} = (0.55) \cdot (\text{exam}) + (0.09) \cdot \text{assignment} + \sum_{i=1}^3 (0.12) \max(\text{exam}, \text{quiz}_i)$$

Office Hours. My office hours for 610 are Thursday after class 1-3. Of course, anyone can drop in and talk to me any time I am free. For greater efficiency, just email me.

Course Outline

- I. *Discrete Time* Stochastic Dynamic Programming (March 5, 7, 12, 14)
 - A. Finite-horizons — application: “The Secretary Problem” (Ross, §I–5)
 - B. Discounted Dynamic Programming (DP): Ross, §I–II
 - Application: Optimal Stopping, and Multi-armed bandits (Ross, §VI, §VII)
 - Weitzman (1979), “Optimal Search for the Best Alternative,” *Econometrica*, 641–54
 - C. Negative DP (Ross, §III) — application: Bayesian sequential analysis; price search
 - D. Positive DP (Ross, §IV) — application: gambling
 - E. Continuous time Poisson DP models with sequential actions
- II. *Continuous Time* Stochastic Dynamic Optimization (March 19 – April 16)
 - A. Stochastic Calculus (≈ 5 lectures)
 1. Brownian motion (O §2, K §3)
 2. Stochastic integration and Ito’s Rule (O §3 – 4, K §4)
 3. Approximating discrete time models [consult Karlin-Taylor, volume II]
 4. Stochastic differential equations (O §5, K §5)
 5. Boundary value problems by tailor-making your own security (O §9, K §6)
 - Finance: Black-Scholes option pricing formula; American v. European options
 - International finance application: “Target Zones & Exchange Rate Dynamics,” Paul Krugman, *QJE*, Vol. 106, No. 3. (Aug., 1991), pp. 669–682.
 6. Dynkin’s formula; Backward/Forward equations; steady-state dist’ns (O §7–8)
 7. Diffusion processes vs. jump processes (K §9)
 - B. Irreversible Decisions, and Optimal Stopping (≈ 3 lectures) (O §10)
 1. Superharmonic/excessive functions (general case, with any number of states)
 2. “Smooth pasting” (scalar case, with two states)
 3. Famous optimal stopping problems in economics (with smooth pasting)
 - I.O: Optimal timing of irreversible investments
 - Macro: Transaction cost (S, s) models; Labour: Wage Search
 - Finance: optimal exercise of put options (O §12)
 - Micro: Gittins Indices
 4. Introduction to filtering — application: Bayesian sequential analysis (O §6)
 5. Barrier policies and the “super-high contact” matching condition
 - C. Real-Time Decision-making, and Optimal Stochastic Control (≈ 1 lecture) (O §11)
 - I.O. (research and development): Giuseppe Moscarini & Lones Smith (2001), “The Optimal Level of Experimentation,” *Econometrica* **69**, 1629–1644.
 - Macro: Dynamic portfolio selection; the linear-quadratic regulation model