

Economics 610 – Quiz #3 (April 12/13, 2001)

- Anyone writing the quiz on April 12 must hand in this sheet to Katya, and not hint to any other Michigan students of its contents.
 - Closed book quiz. You have 50 minutes. Justify what you claim.
 - Problem 1 counts also half towards the assignment 1 grade, even if this quiz does not count towards your final grade.
 - Below, $\langle W_t \rangle$ is always a standard Wiener process.
 - The formulation of your solution is the single most important step. No problem here is hard if you first write it down with a little forethought. Use your economic intuition. Start right, and think somewhat, and explain it to yourself. Only then should you solve it. Then ex post, use your intuition: Are you making sense?
 - Have fun: After all, if the quiz hurts your grade, it doesn't count. Besides, you'll do better without getting stressed out.
1. Assume that an impatient price searcher (discount factor $0 < \delta < 1$) is looking for a pipe wrench that is worth 1 to him. He must pay $c \in (0, 1)$ per search. He pays the search cost up front each period, and if he buys at price p gets payoff $1 - p$, discounted next period. Prices are distributed independently and uniformly on $(0, 1)$. He wishes to maximize the expected present discounted value of his expected *surplus*, namely the final consumer surplus plus the sum of search costs, all appropriately discounted.
 - (a) Precisely describe his behavior (i.e. give an exact quantitative recipe for him to follow) in a city like Chicago, which is so large as to have essentially an infinite number of stores. [8]
 - (b) Graph your results in consumer surplus space. [2]
 - (c) [Bonus] Repeat part (a) for a small city like Ann Arbor, with only n stores. Draw pertinent contrasts. [5]
 2.
 - (a) Solve the stochastic differential equation $dX_t = 3X_t^{2/3}dW_t + 3X_t^{1/3}dt$. [6]
 - (b) If $X_0 = 0$, what is the chance that $X_3 > 0$? [2]
 3. A stock price X_t follows a geometric Brownian motion $dX_t = X_t(\mu dt + \sigma dW_t)$, where μ and σ are constants. You discount the future at interest rate $r > 0$.
 - (a) Given the current price x , what is a *nonexpiring* put option worth that allows you to optimally sell a share of the stock *at any time* at price p ? [13]
 - (b) As a function of the price, is the put option price increasing or decreasing in x ? Convex or concave? Precisely graph your results. [4]